

# *Introduction to Fuzzy Mathematics*

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# Fuzzy numbers

**Fuzzy numbers are fuzzy sets defined over the set of real numbers, which model our concept of approximate value.**

**Constraints:**

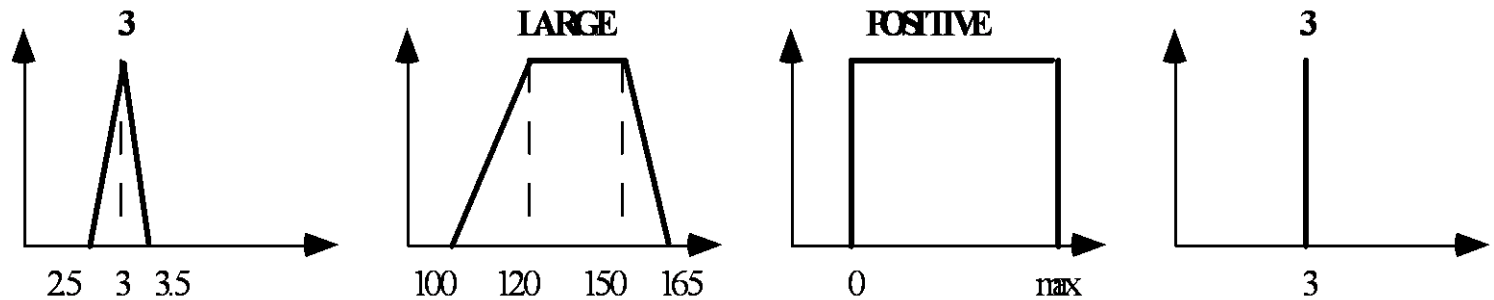
- 1. normal fuzzy sets**
- 2. convex fuzzy sets (all  $\alpha$ -cut intervals should be closed)**
- 3. the support should be bounded**

**Constraint 1. captures the concept of approximate value corresponding to a number**

**Constraints 2. and 3. are needed to define arithmetics**

# Unifying role of fuzzy numbers

The same formalism may be used for fuzzy numbers, crisp numbers, intervals and qualitative values



# Arithmetics of fuzzy numbers

Arithmetics on fuzzy numbers is based on two properties:

1. each fuzzy number can be completely represented by its  $\alpha$ -cuts, in a unique way
2. the  $\alpha$ -cuts of fuzzy numbers are closed intervals of real numbers

# The four main operations

The four operations are defined as the union of the operations on intervals ( $\alpha$ -cut) that compose the fuzzy number

$$[a, b] + [d, e] = [a+d, b+e]$$

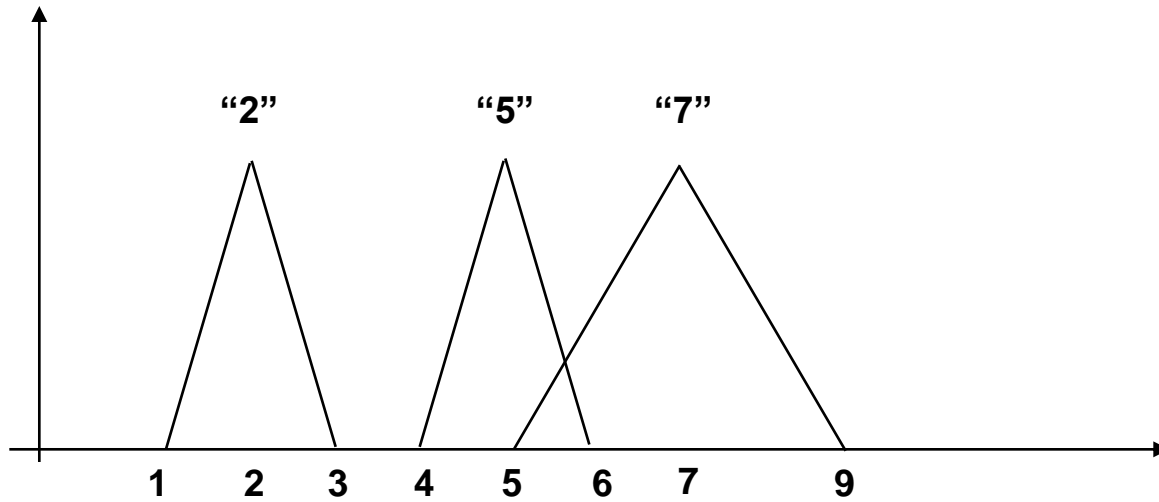
$$[a, b] - [d, e] = [a-e, b-d]$$

$$[a, b] * [d, e] = [\min(ad, ae, bd, be), \max(ad, ae, bd, be)]$$

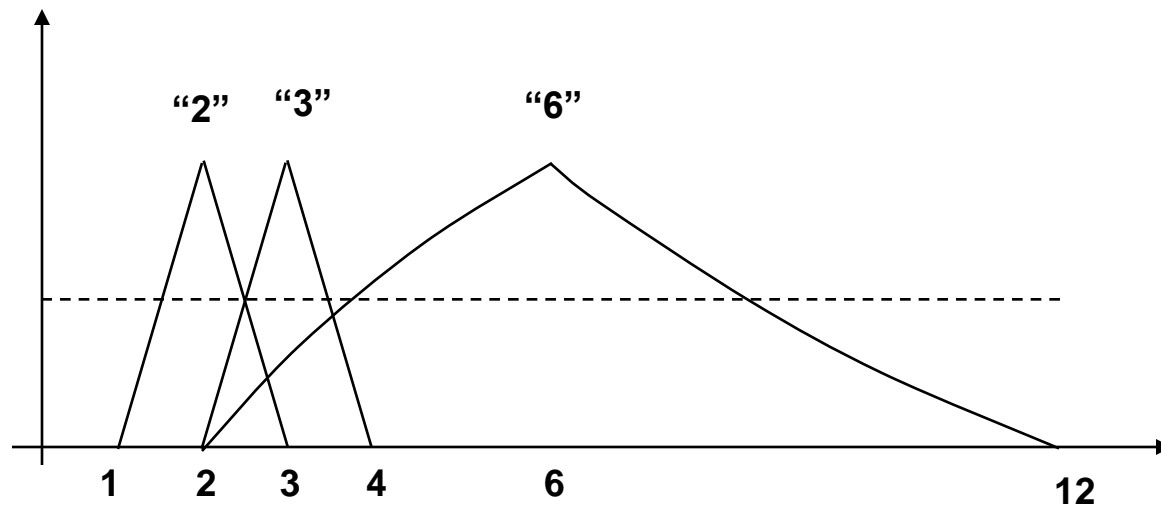
and, if  $0 \notin [d, e]$

$$\begin{aligned} [a, b] / [d, e] &= [a, b] * [1/d, 1/e] \\ &= [\min(a/d, a/e, b/d, b/e), \max(a/d, a/e, b/d, b/e)] \end{aligned}$$

# Sum

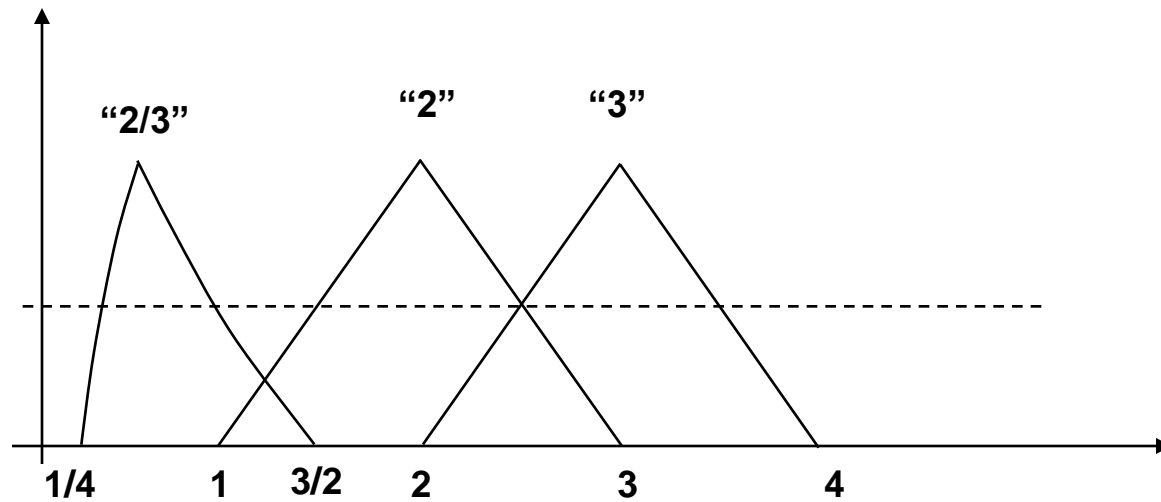


# Product





# Division



## ... and so on

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It is possible to define

- fuzzy functions,
- fuzzy integrals,
- fuzzy derivatives
- ...

# Applications

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Fuzzy numbers to represent approximation.

Fuzzy mathematics to implement models with fuzzy numbers, possibly integrating them with interval and crisp numbers.

# An example: fuzzy simulation

Monitoring for preventive diagnosis on a packaging machine.

The problem: identify the need of maintenance before malfunctioning

The proposed solution (Bonarini, Bontempi, 1994): simulate the dynamical model of the critical mechanism and match it with the actual data and the desired boundaries to understand whether the present behavior might be considered as “regular” or close to “malfunctioning”.

The model is based on parameters that can be only approximately be estimated: fuzzy simulation.

