Fuzzy rules

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What is an inference rule?

We can consider a rule as a **model**, a way to define a **mapping** from input to output.

We consider forward chaining rules, having the shape

IF <antecedent> THEN <consequent>

where:

- <antecedent> is a set of <clause>s related by logical operators AND, OR, NOT
- <consequent> is a set of <clause>s related by the logical operator AND
- <clause> in general is either a <proposition>, a sequence of symbols, or a <pattern> a sequence of symbols and variables

What are rules for?

Rules are used to represent inferential relationships among pieces of knowledge.

They are used to implement Knowledge-Based Systems, among which Expert Systems are mostly known as successful AI applications, e.g. for diagnosis, forecast, design, etc.

Example of rules

Mapping among propositions

IF (apple is red) THEN (apple is ready)

Mapping among patterns

IF (?a is apple) AND (?a is red) THEN (?a is ready)

Why we use rules?

Rules are used to infer new knowledge from known facts.

E.g.:

From

(apple1 is red) and **(apple1 is apple)**

and the rule

IF (?a is apple) AND (?a is red) THEN (?a is ready)

we can infer

(apple1 is ready)

How do we use rules?

1. **Pattern matching**: identify those rules whose antecedent match the known facts (fact base). These can be considered for activation, given the corresponding assignment to variables

2. **Select the rule(s) to be activated**: among the rules and activation environments identified at step 1, select the rules that should be activated, i.e., whose consequents have to be asserted as new knowledge in the fact base

3. **Activate the selected rules**: assert the consequents of the selected rules.

What is a fuzzy rule?

A **fuzzy rule** is a rule whose clauses have the shape

(V is L)

where V is a linguistic variable and L is a label, a value for V associated to a fuzzy set. This is a **linguistic clause**.

Usually, clauses in the antecedent are only related by the AND operator.

The antecedent is usually **matched** against facts that are represented as values of real-valued variables corresponding to the linguistic variables

The consequent may be one of two types …

Linguistic rules

Linguistic rules (Mamdani): the consequent is a conjunction of linguistic clauses

IF (A is LA_n) AND (B is LB_k) AND... THEN (U is LU_m) AND ...

E.g.:

IF (Distance is Far) AND (BallDirection is Front)
THEN (Speed id High) AND (Direction is Ahead)

This can be considered as a mapping between
the **interpretation of an input configuration**
and
a **symbolic description of the desired output**

Model rules

Model rules (Sugeno, or Takagi-Sugeno-Kosko TSK):

bind a **model** (linear, non linear, NN, …) to the linguistic interpretation of its **applicability conditions**

IF (A is LA_n) AND (B is LB_k) AND… THEN U is f(*A*, *B*)

E.g.:

IF (Temperature is High) AND (Pressure is High)

THEN Heating = 2000 - 3*T* -7*P*

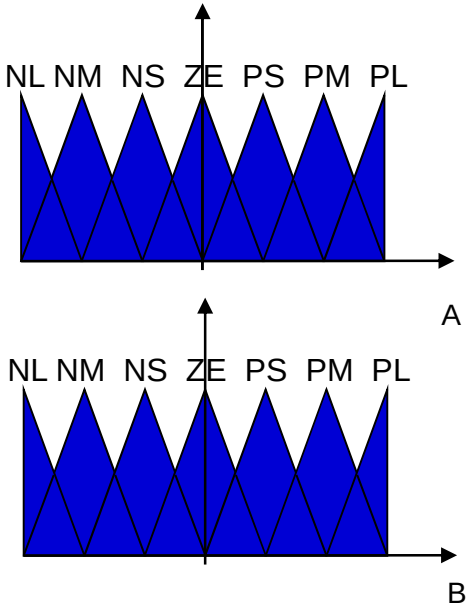
This can be considered as a mapping between

the **interpretation of an input configuration (the applicability condition of a model)**

and

a **model to be applied to the input real values to obtain the output**

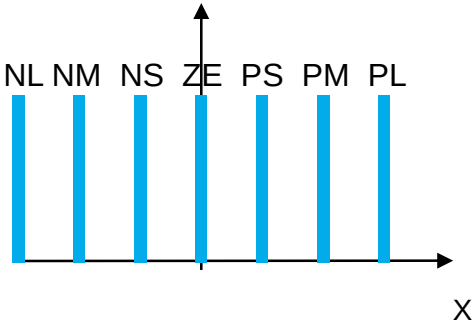
How to use fuzzy rules (Mamdani)



R1: if A is PL and B is PS Then X is PM

R2: if A is PM and B is PS Then X is PS

R3: if A is PL and B is PM Then X is PM



Input matching

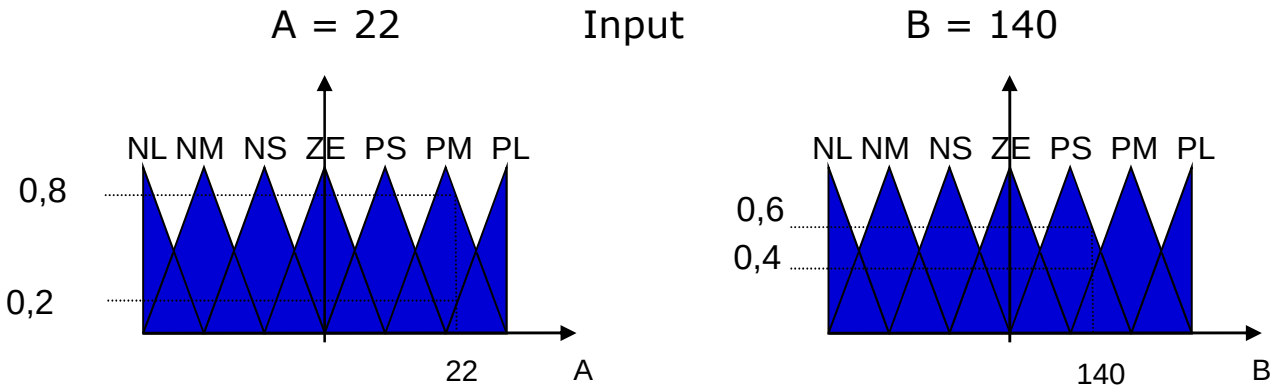
Combination of matching degrees

Eventual combination with rule weight

Aggregation of output from different rules

Eventual defuzzyfication of output

Input matching



R1: IF (A is PL) (B is PS) THEN (X is PM)

0,2 0,6

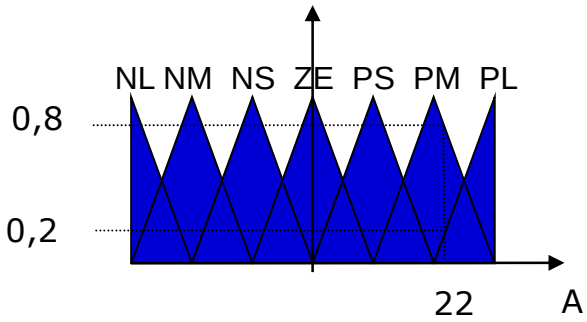
R2: IF (A is PM) (B is PS) THEN (X is PS)

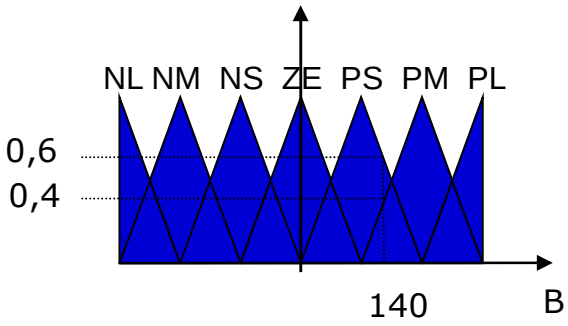
0,8 0,6

R3: IF (A is PM) (B is PM) THEN (X is PM)

0,8 0,4

Combination of matching degrees in the antecedent





We use the **min** operator

R1: IF (A is PL) (B is PS) THEN (X is PM)

0,2 0,6

0,2

R2: IF (A is PM) (B is PS) THEN (X is PS)

0,8 0,6

0,6

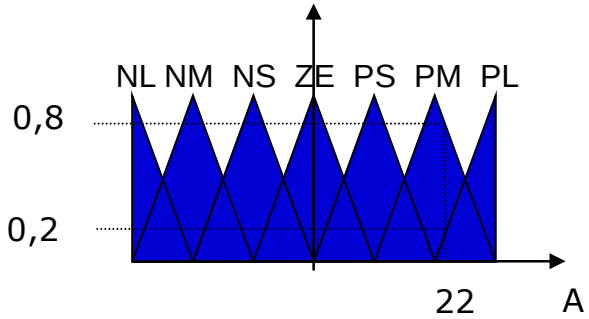
R3: IF (A is PM) (B is PM) THEN (X is PM)

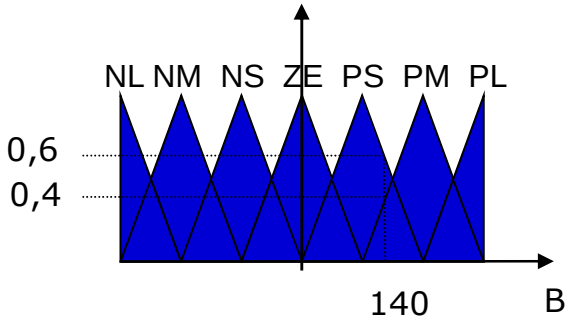
0,8 0,4

0,4

The matching degree of the antecedent is a measure of how the rule fits the current situation

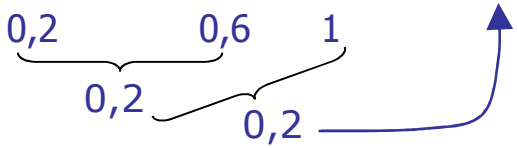
Combination with the rule weight



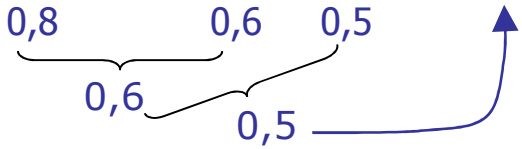


We use the **min** operator

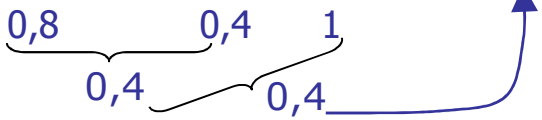
R1: IF (A is PL) (B is PS) THEN (X is PM)



R2: IF (A is PM) (B is PS) THEN (X is PS)



R3: IF (A is PM) (B is PM) THEN (X is PM)



The resulting value is a measure of how much the rule is "good" given the situation and "per se"

Output aggregation

R1: if (A is PL) (B is PS) Then (X is PM)

0,2

R2: if (A is PM) (B is PS) Then (X is PS)

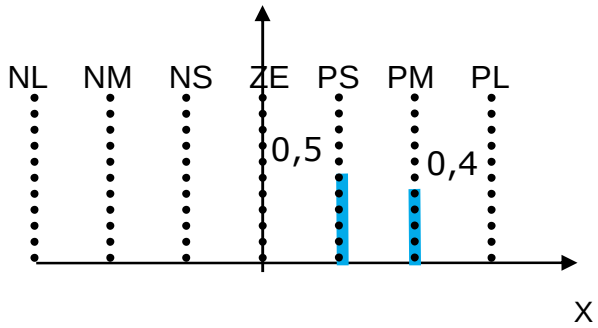
0,5

R3: if (A is PL) (B is PM) Then (X is PM)

0,4

X is PM with weight 0,4 ← **max**

X is PS with weight 0,5

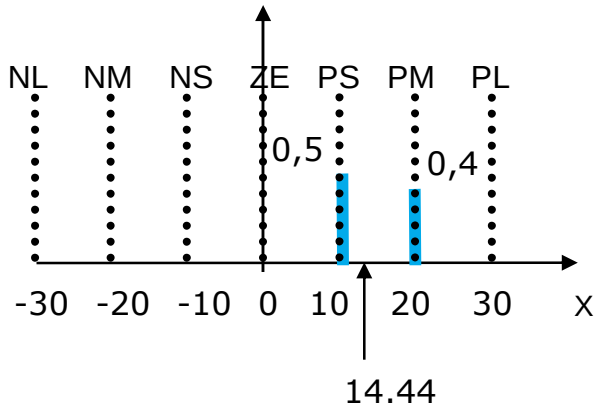


We use the operator **max** to aggregate weights given to the same output value

(Eventual) defuzzyfication

X is PM with weight 0,4

X is PS with weight 0,5



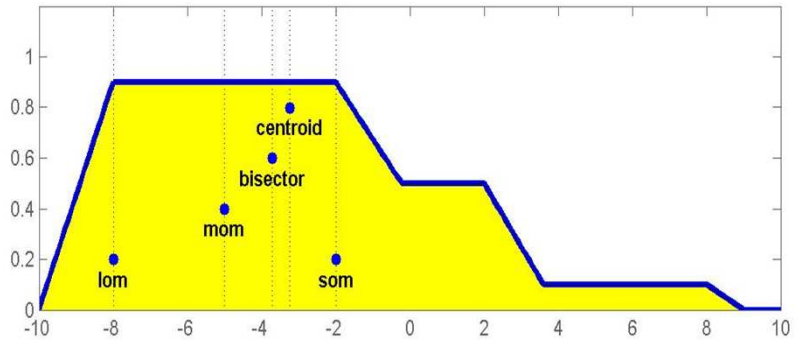
(10*0,5 + 20*0,4)/(0,5 + 0,4) = **14.44**

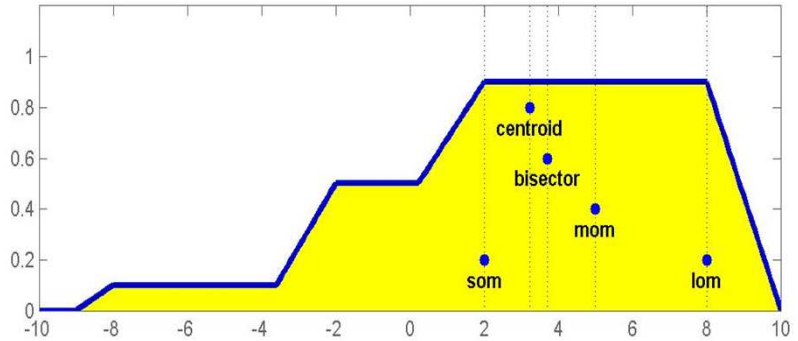
We use the operator "weighted media" on the weights of the output values to obtain a numerical values

Defuzzyfication

Also for the defuzzyfication we have many possible operators

- Centroid
- Bisector
- Average of maxima
- Lowest maximum
- Highest maximum
- Center of the highest area
- ...

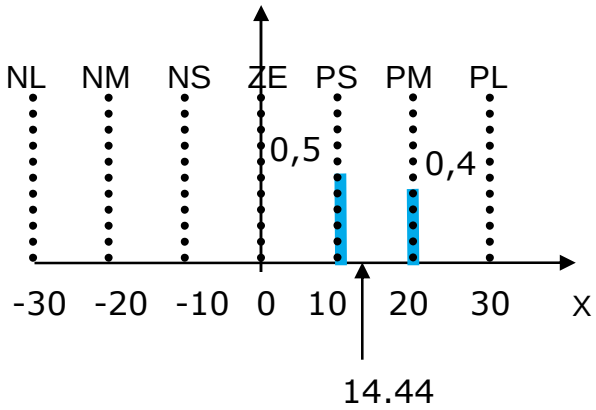




Another possible output: linguistic approximation

X is PM with weight 0,4

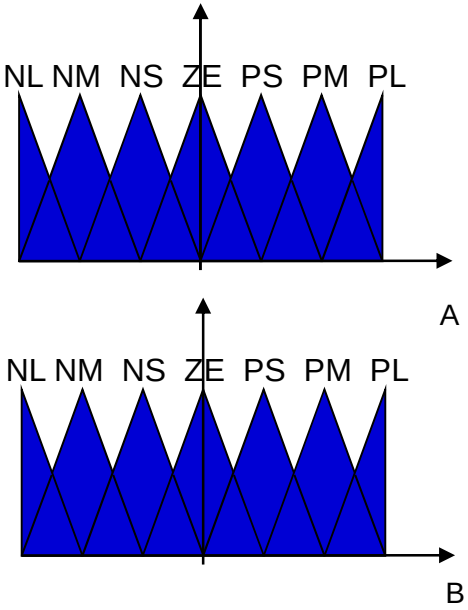
X is PS with weight 0,5



(10*0,5 + 20*0,4)/(0,5 + 0,4) = **14.44**

...so **X is PS**

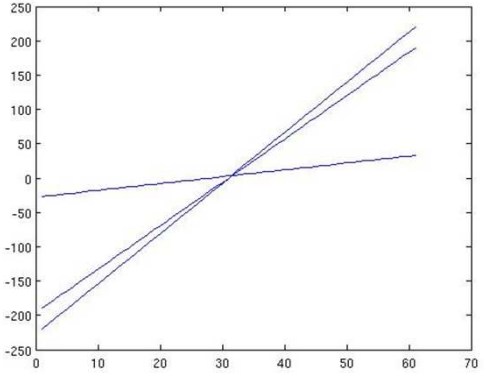
How to use fuzzy rules (Sugeno)



R1: if A is PL and B is PS Then X is A+2B

R2: if A is PM and B is PS Then X is A +3

R3: if A is PL and B is PM Then X is A+B



Input matching

Combination of matching degrees

Eventual combination with rule weight

Aggregation of output from different rules

Eventual defuzzyfication of output

Same as with Mamdani

Output aggregation

R1: if (A is PL) (B is PS) Then (X is A+2B)

0.2

R2: if (A is PM) (B is PS) Then (X is A+3)

0.5

R3: if (A is PL) (B is PM) Then (X is A+B)

0.4

X is (0.2*(A+2B)+0.5*(A+3)+0.4*(A+B))/(0.2+0.5+0.4)

Since A=22 and B=140 then X=125.18

This is a weighted combination of the models expressed in the output of the rules matching the inputs

Some exercises

Define the rules to control the light in your room according to the external light coming from a window

Define the rules to decide what to do when you see a large, bad-looking dog on your way