

Marine Robotics

Unmanned Autonomous Vehicles in Air Land and Sea

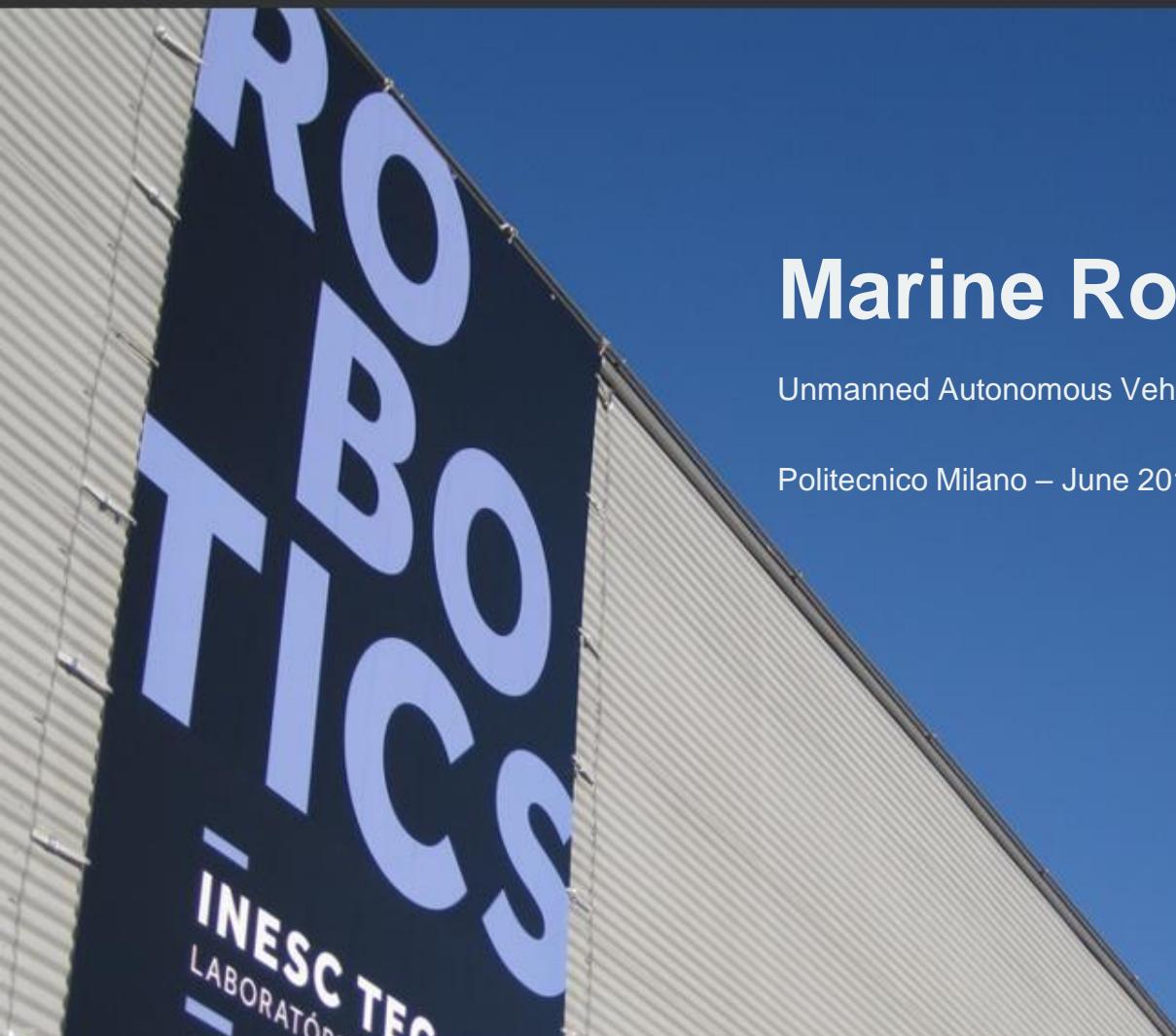
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Sensors and Perception





Perception

Sensor – dispositivo that maps environmental attribute in a quantitative measure

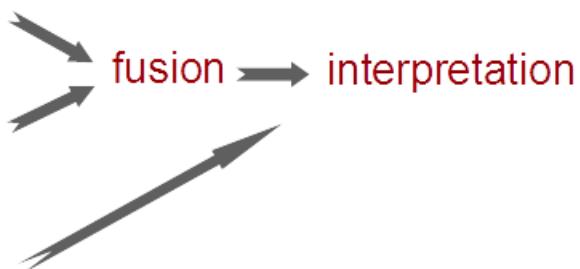
Sensors convert energy from one form to another (transducers)

Perception – interpretation and fusion of sensor measurements,
“knowledge” of the environment (external/internal)

sensor → acquisition → filtering/processing

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“sensing” 
*increasing abstraction
decreasing granularity*

Acquisition

- Can be performed at different points in the processing chain
 - one can have analog processing before A/D conversion
- Obtaining measures in a convenient format
 - Analog/Digital conversion
- Communication with a specific sensor, data reception

Filtering /pre-processing

- Filtering – noise reduction
- “outliers” elimination
- Selection of frequency range of interest
- Recalibration
- Changing of data representation
 - ex: coordinate transformation
- Basic processing
 - ex: vision segmentation
- Depending on the sensor

Sensor fusion

In general it is necessary to combine information from multiple sensors

- Noise
- Limited accuracy and precision
- Reliability (redundancy)
- **Limited perception of the environment**
 - Incomplete description (types of measures, occlusions ...)
- Cost
 - It can be more efficient combine multiple sensors than to use a more expensive one

Sensor fusion – Combine multiple sensor measures in coherent information

Sensorial integration – Use information from multiple sensors to do something useful



Sensor fusion

- Combine data from different sources
 - Multiple sensors
 - Different physical space
 - Different times
- Methods incorporating uncertainty in the sources
 - Discrete probabilistic filters
 - Neural networks
 - Kalman filtering (EKF, UKF etc)
- Coherent result – “virtual” sensor data

Interpretation

- Depends on the task
- Usually requires a previous environment model (“match”)
- Extraction/detection of relevant information from data (maximum, minimum, temporal events)
- Higher abstraction building (ex: topological localization)
- Clustering, pattern recognition , machine learning

Sensor Classification

- Proprioceptor/exteroceptor
 - Proprioceptor – internal state measurement (battery level, wheel velocity, etc)
 - Exteroceptor – external quantity measurement, environmental (distance to objects, external temperature etc)
- Active / Passive
 - Active – energy emitting to the environment (sonar, radar)
 - Passive – only receive energy: vision
- Contact / Contactless
- With or without physical contact

Perception sensors in marine environment



Radar

- Reflection of radio pulses and microwaves
- Obstacle detection and mapping
- Widely used in marine navigation
- Large dimension and weight
- Sensitive to atmospheric conditions
- Multiple levels of information (from simple echo's and target tracking)

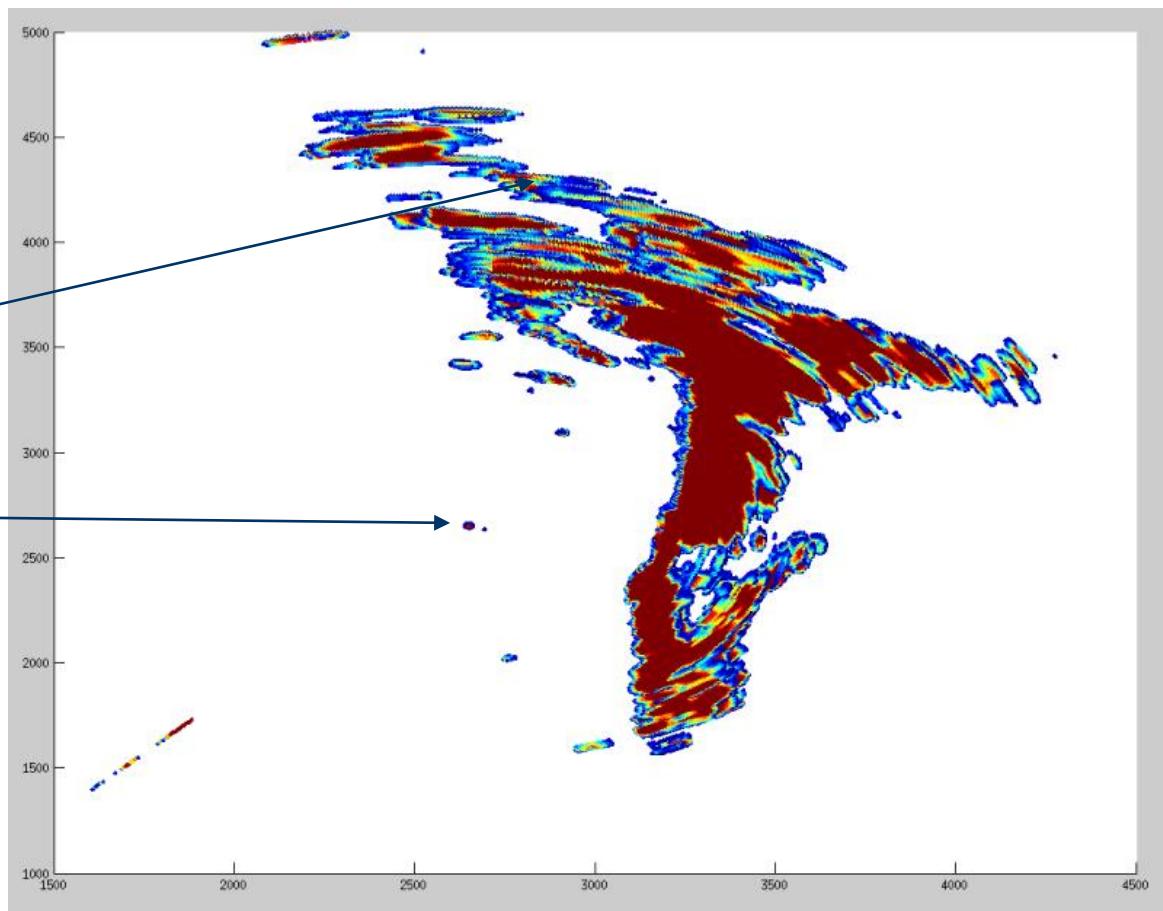


Radar

Echo return of L shaped pier
from RADAR mounted on ROAZ
ASV (in La Spezia, Italy)

Visible returns from boats
stationed behind pier

ROAZ (Radar center)



Underwater sonar sensors





- Ranging/bathymetric
 - Altimeter
 - Subbottom profiler
 - Sonar profiler (mechanic)
 - Multibeam sonar
 - SWATH bathymetric sonar (interferometric)
- Imaging
 - FLS
 - Sidescan sonar
 - 3D imaging/ranging (CodaOctopus Echoscope)
- ADCP / DVL



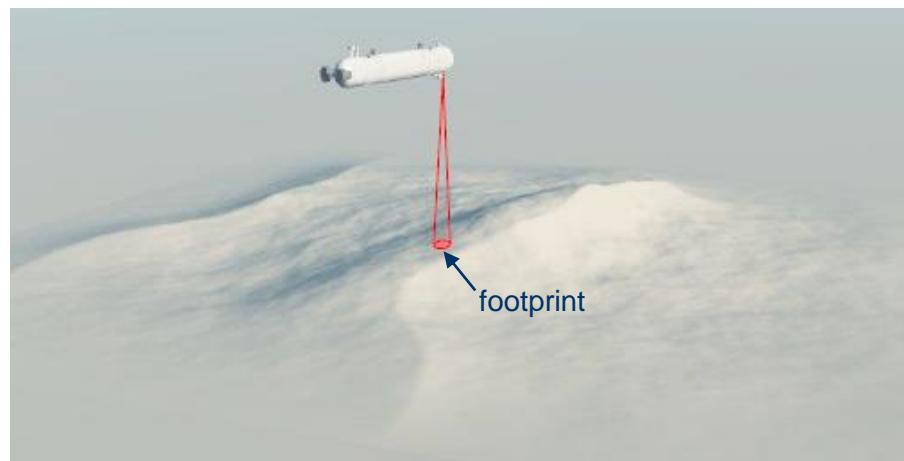
Underwater acoustics

- Sound – primary source of information for sensing underwater
- Underwater acoustics depends on multiple factors
 - Sound velocity – salinity, temperature
 - Multipath, shallow water and reflections on surface
 - Ambient noise
 - Interference



Single Beam Echosounders

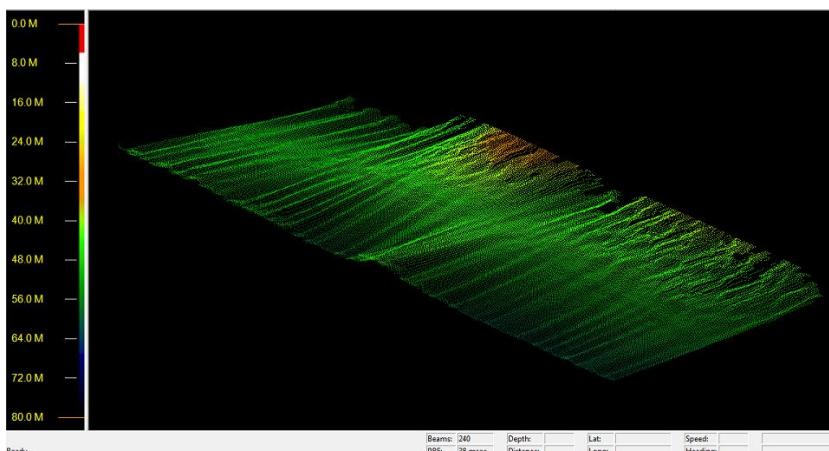
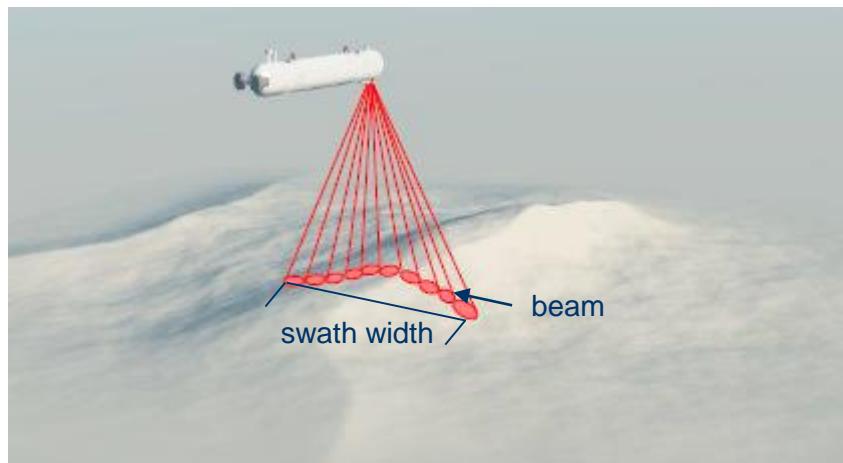
- Sonar emits a pulse and detects echo from target, range determined by time of flight
- Range
 - FAT (First return Above Threshold)
 - Maximum return
 - Echo profile (range binning)
- Used for depth soundings or for obstacle ranging (limited application)
- Single narrow beam (usually less than 3°)
- Integrated AHRS (Heading, Pitch, Roll)
- Small size
- Typically with serial interface



[ISA500]
www.ImpactSubsea.com

Multibeam echosounder

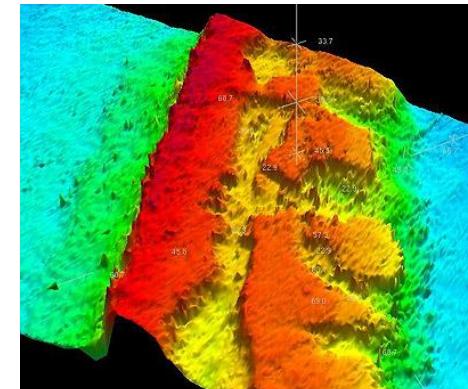
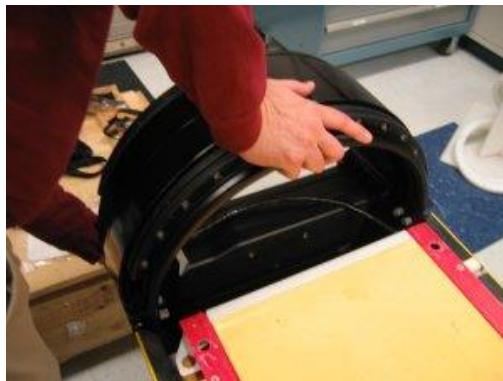
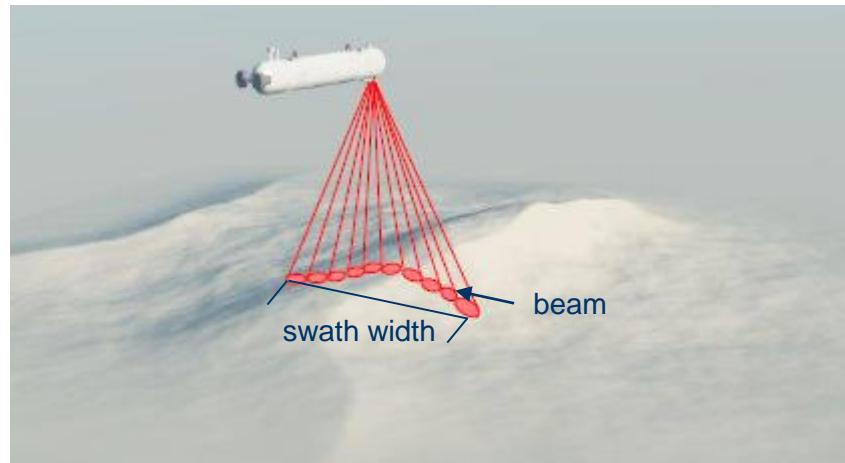
- Array of transducers emits and receives a fan shape beam to the seabed
- Acquire depth scans, measuring the signal travel time along multiple directions
- Can also return the echo profile in each direction (with binning)
- Each beam varies from 0.5 to 3°^a
- Price highly dependent on angular and range resolution (from ~30k€ for 3° to 100/200 K€ for 0.5)





Multibeam echosounder

- All beams grabbed simultaneously
- Resolution dependent on slant range
- Higher the altitude the lower resolution
- Higher frequencies give higher resolution but lower range
- Frequencies ranging from 500kHz to 2.25MHz





Multibeam Sonar

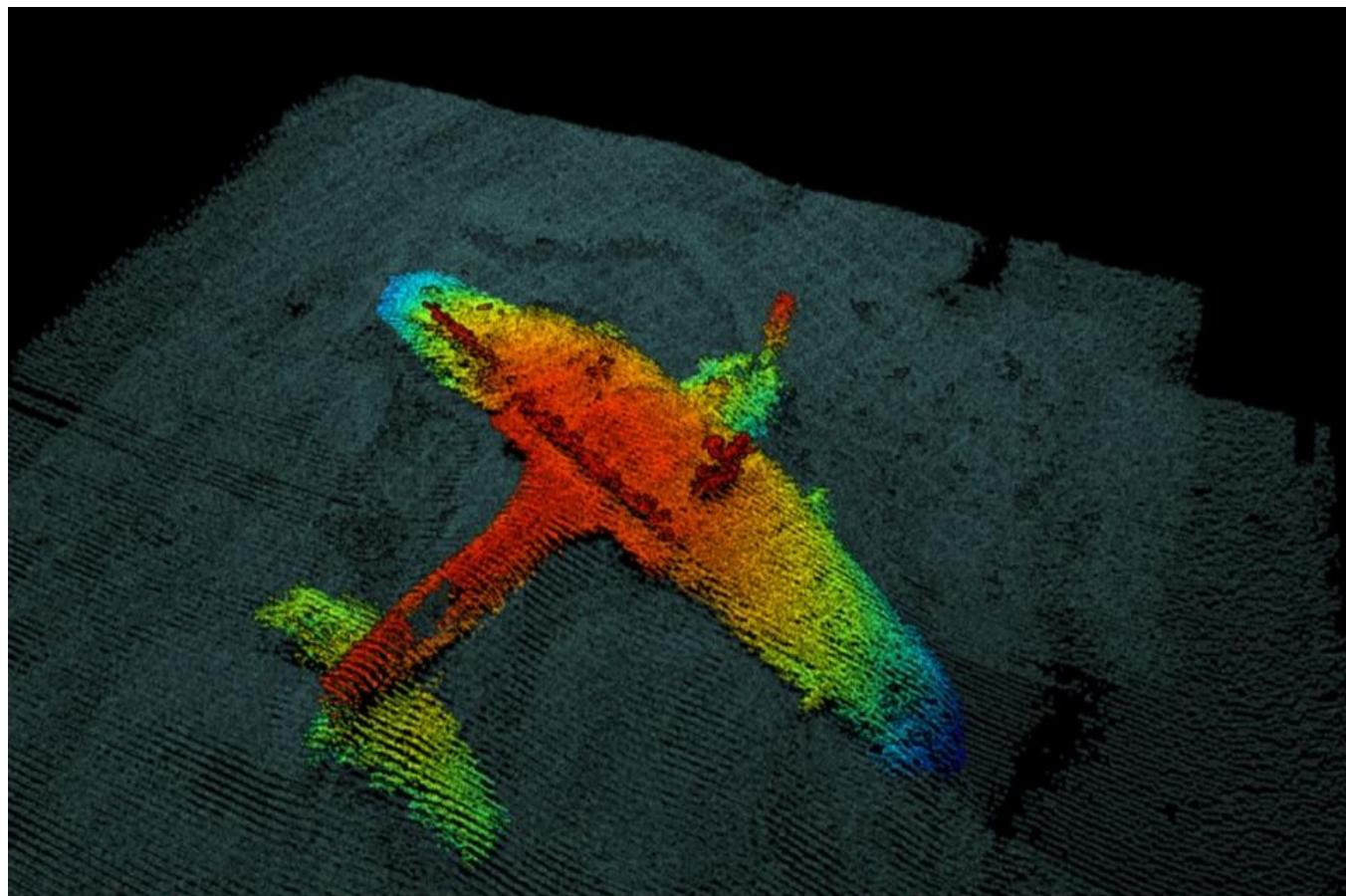
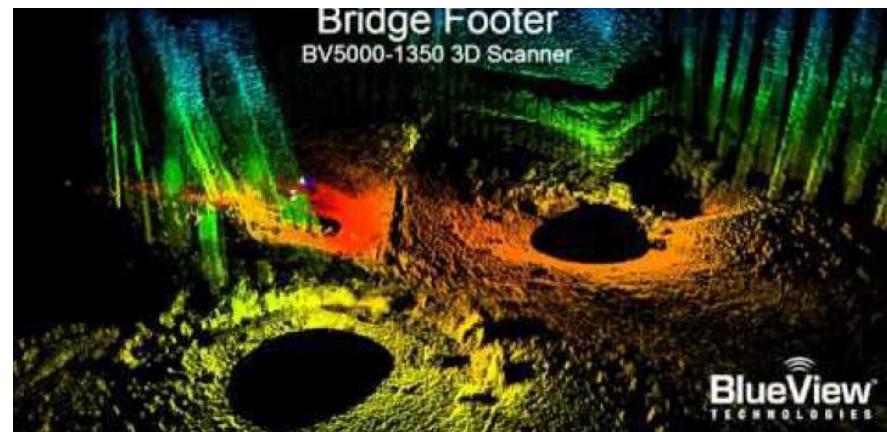


Image from Teledyne Blueview (MB2250)

Multibeam echosounder

- Depending on the required mission can be mounted either vertically or horizontally.
- Common arrangement looking down transverse to vehicle direction of motion to provide bathymetry data
- Different points on the sea floor provide returns at different angles
- Mechanically rotating solutions (for fixed position) with vertical beam provide wide horizontal coverage
 - Ex: [Blueview BV5000](#)



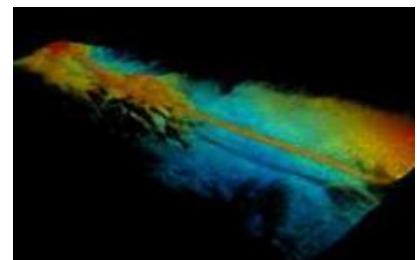
Blueview BV5000

Images from Teledyne Blueview



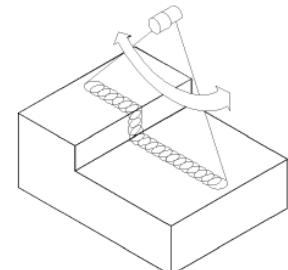
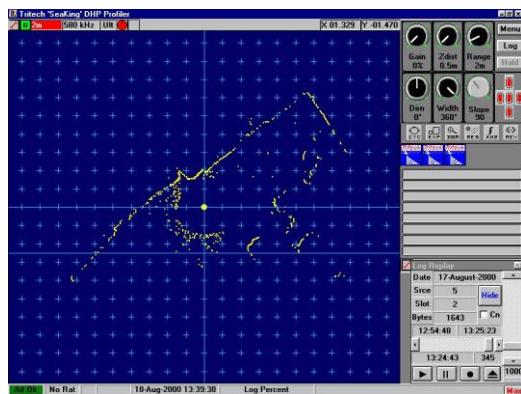
Electronic actuated multibeam

- Multibeam with the possibility of electronically controlling the vertical angle of the beam pattern
- Solid state tilting
- Provides almost 3D by sweeping the vertical angle fast
- Currently there is no commercial solution available (Tritech Eclipse no longer in production)



Mechanically rotating pencil beam profiler

- Rotating narrow beam
- Similar to a single echosounder but with a mechanical rotation of the transducer
- Narrow beam 1°-2°
- For distance measurement and profilling



Images from Imagenex
www.imagenex.com

PENCIL SHAPED SONAR BEAM
SCANS IN A VERTICAL PLANE
TO MEASURE BOTTOM PROFILE



Forward Looking Sonar (FLS)

- Imaging sonar
- Sonar “illuminates” the environment and receives multiple vertical wide and horizontal narrow beams
- For each beam a complete echo profile is returned
- All beams acquired simultaneously
- Real-time imagery
- Applications: Search and Rescue (SAR), Obstacle Avoidance, Target Tracking and Subsea Monitoring and Inspection

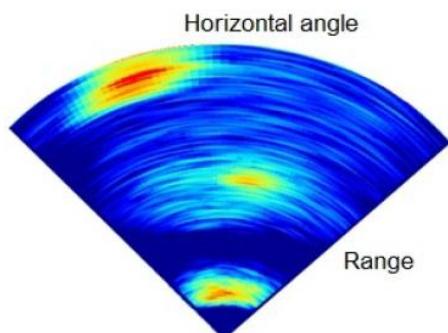
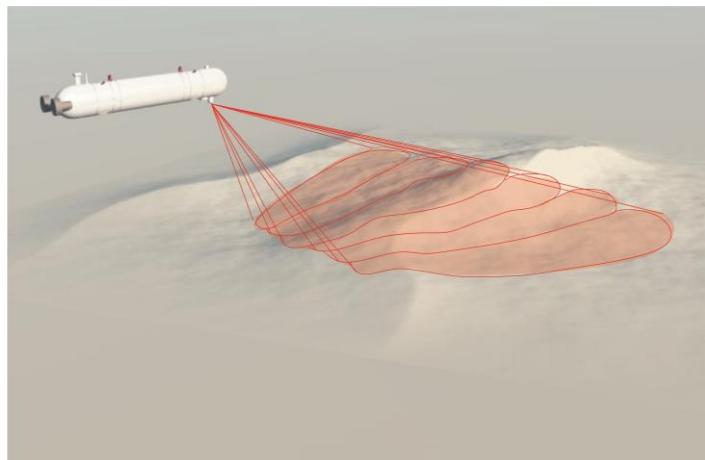
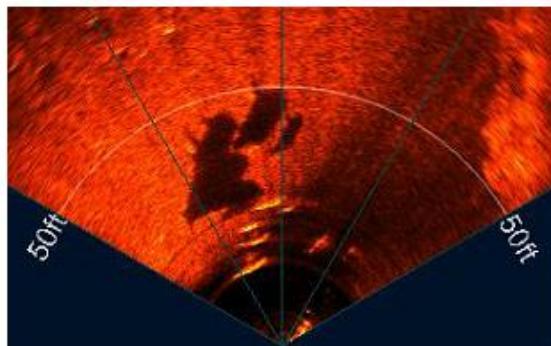


Image from Far Sounder
www.farsounder.com.uk

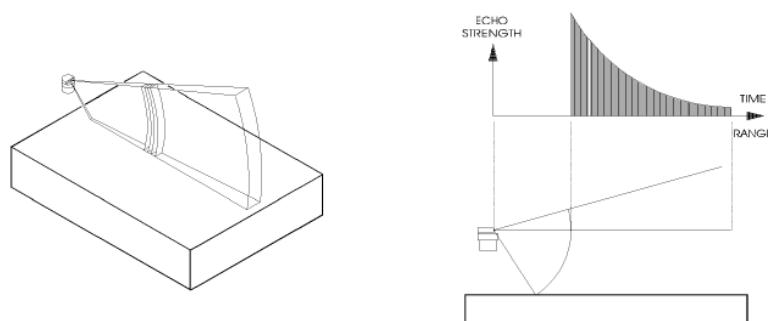


[Germini 720i]
www.germini720i.com
www.tritech.co.uk

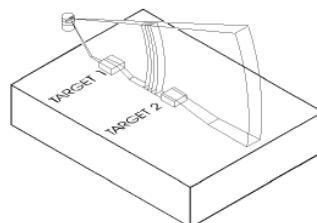


Forward Looking Sonar (FLS)

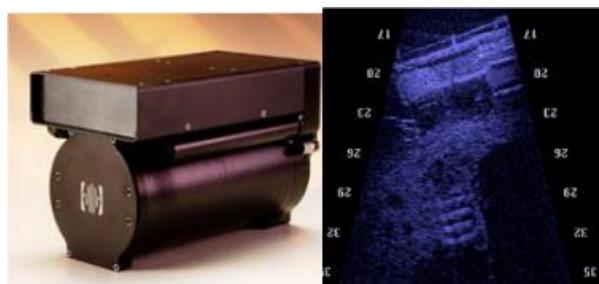
- FLS does not provide distance information
- Return echo at a given distance (time delay) can be anywhere within the in the wide angle of the beam
- Can also be mounted in vertical configuration (ex: for lateral looking)



ECHO STRENGTH VS TIME WHEN
FAN SHAPED SONAR BEAM
INTERSECTS WITH A FLAT BOTTOM



FAN SHAPED SONAR BEAM
INTERSECTS WITH A FLAT BOTTOM
AND TARGETS



Didson sonar from Soundmetrics

Images from Imagenex
www.imagenex.com

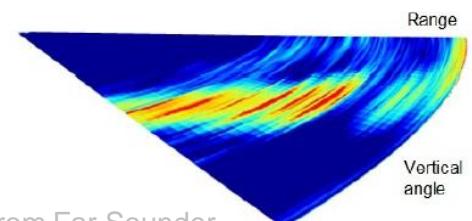
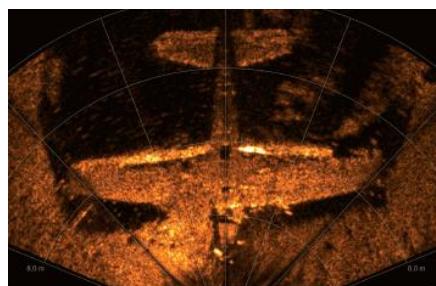
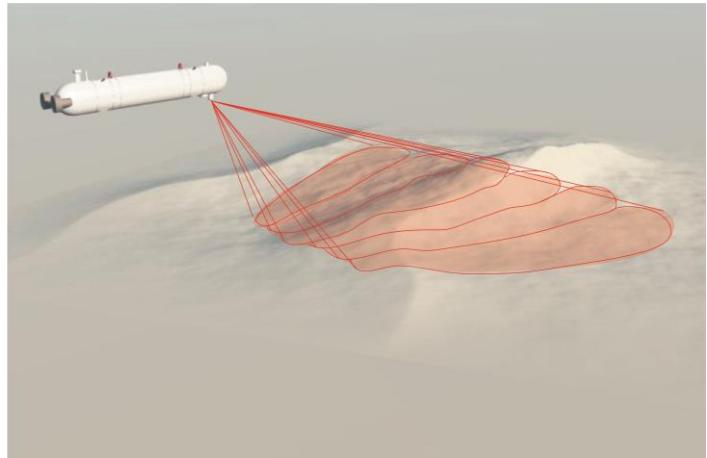


Image from Far Sounder
www.farsounder.com.uk

Forward Looking Sonar (FLS)

- Single or double frequency
- High quality imaging from higher frequencies (ex: Aris Explorer 3000 at 3MHz)
- Range resolutions up to 3mm
- Ranges from 5m (Aris 3000) to 100m(BV M900)
- Beam width typically 1° (or less)
- Horizontal span from 45° to 90°
- Models with two crossing beam patterns (for AUV installation)
- Typical example Tritech 720i
 - 720 kHz operating frequency
 - Number of Beams: 256
 - Swathe: 120°
 - Vertical Beamwidth: 20°
 - Range Resolution: 8mm



Teledyne Blueview M900



Mechanically scanned imaging sonar (MSIS)

- Mechanically rotated transducer that scans a 2D area
- Low scanning rate
- Vehicle motion produces distortion in scan (different directions taken at different times)
- Good for obstacle detection
- Example Tritech MSIS
 - Vertical wide beam (20° - 40°)
 - Narrow horizontal beam (1.5 - 3°)
 - Continuous 360° rotation with 0.45° of mechanical resolution
 - Range: 100 m (650kHz) – 300 m (325kHz)

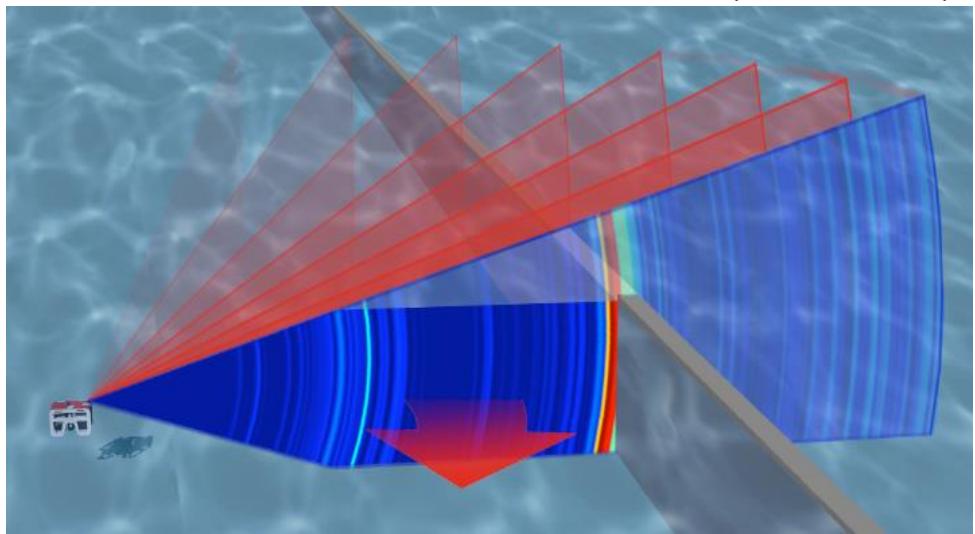


Image from [1]



[Super Seaking]
www.tritech.co.uk

[1] David Ribas, "Underwater SLAM fro structured environment using an imaging sonar", Ph.D. Thesis, Univ. Girona, 2008



360° scan in a structured environment

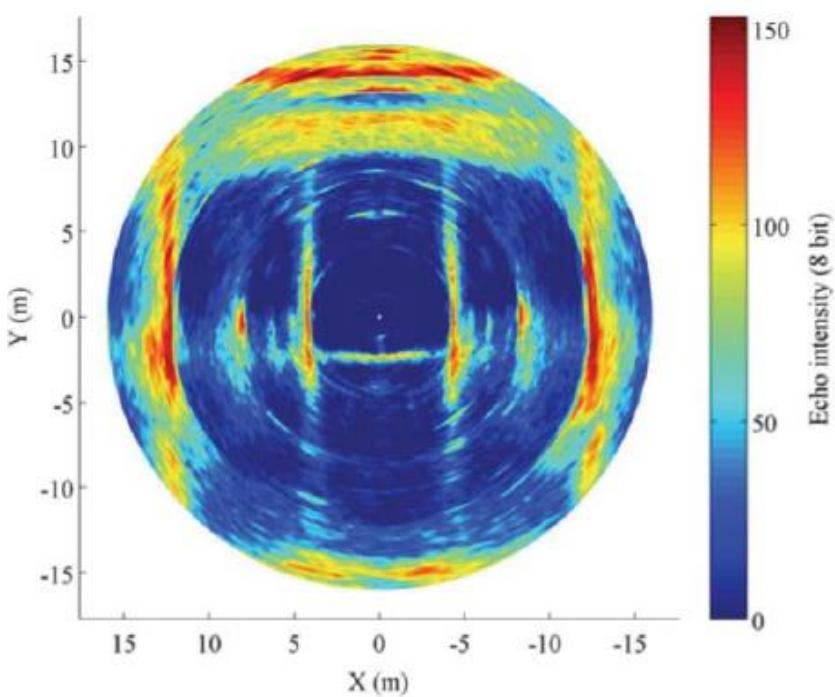
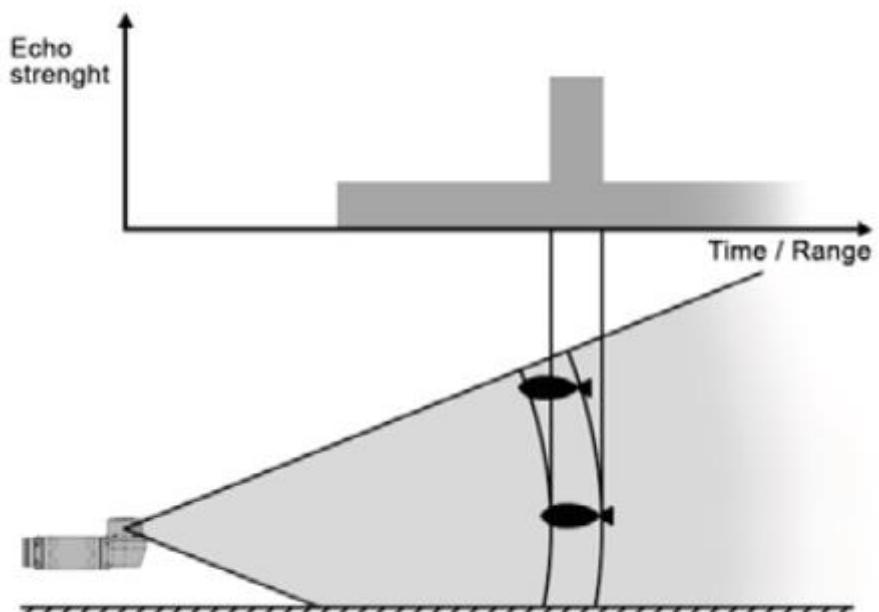


Image from [1]



Motion distortion

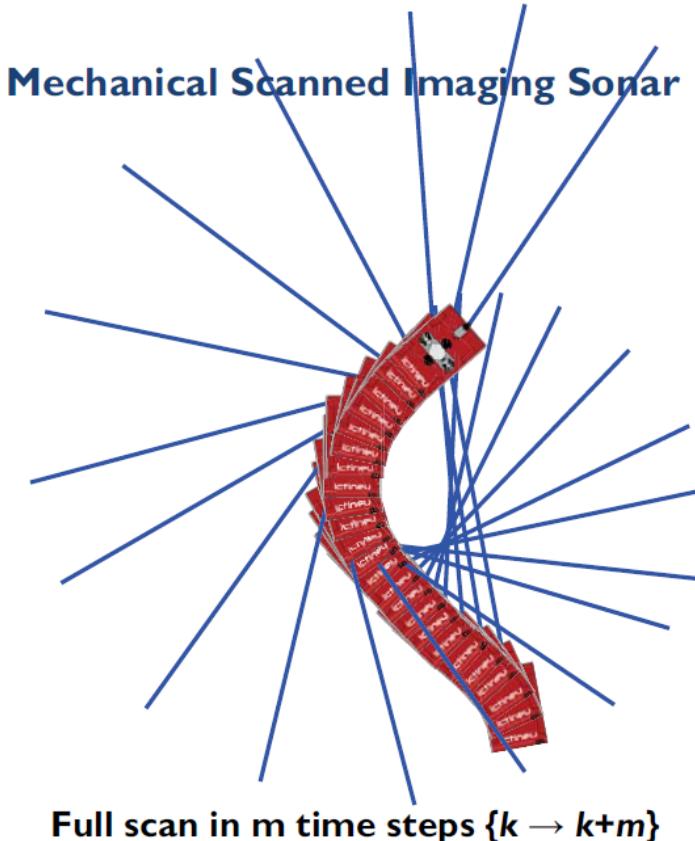
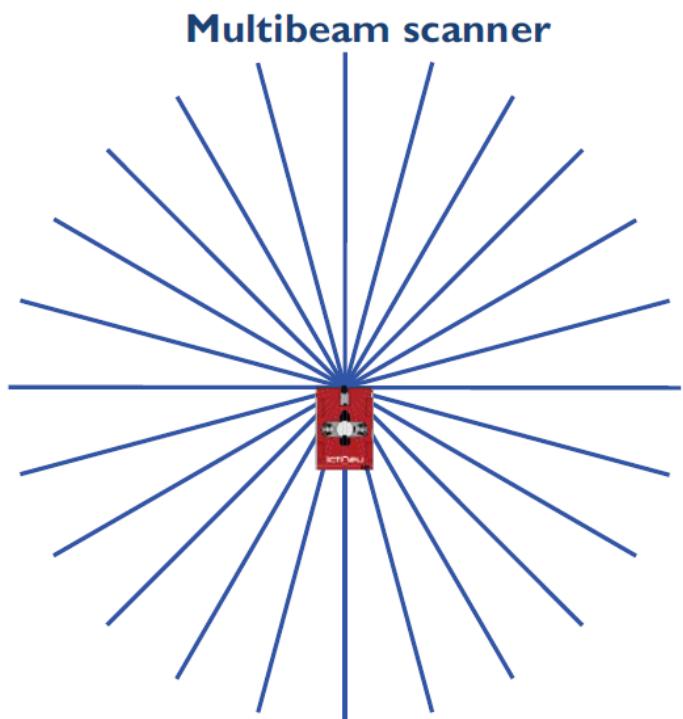
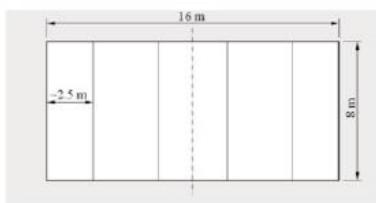


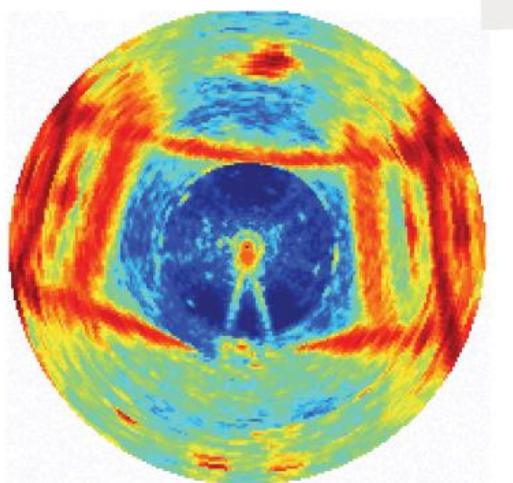
Image from P. Ridao "An introduction to Applied Underwater Robotics", Bts 09



Motion distortion



Distorted Data



Corrected Data

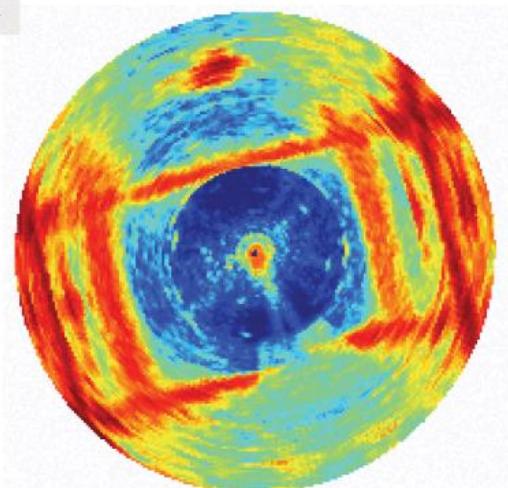


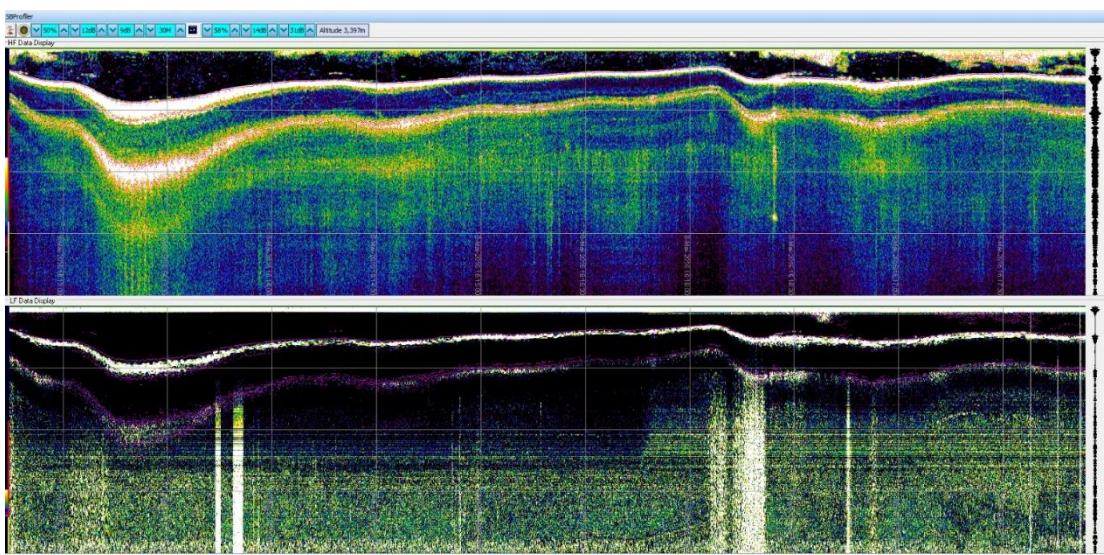
Image from P. Ridao "An introduction to Applied Underwater Robotics", Bts 09

Subbottom profilers

- Echo profile from material frontiers under the sea bottom
- System is capable of penetrating the seabed and highlighting structural differences that are hidden from view
- Applications: Site survey, Route survey, Pipeline crossing, Wreck search and Object detection;
- Seaking beamwidth: 4.5° (20kHz) and 4° (200kHz)

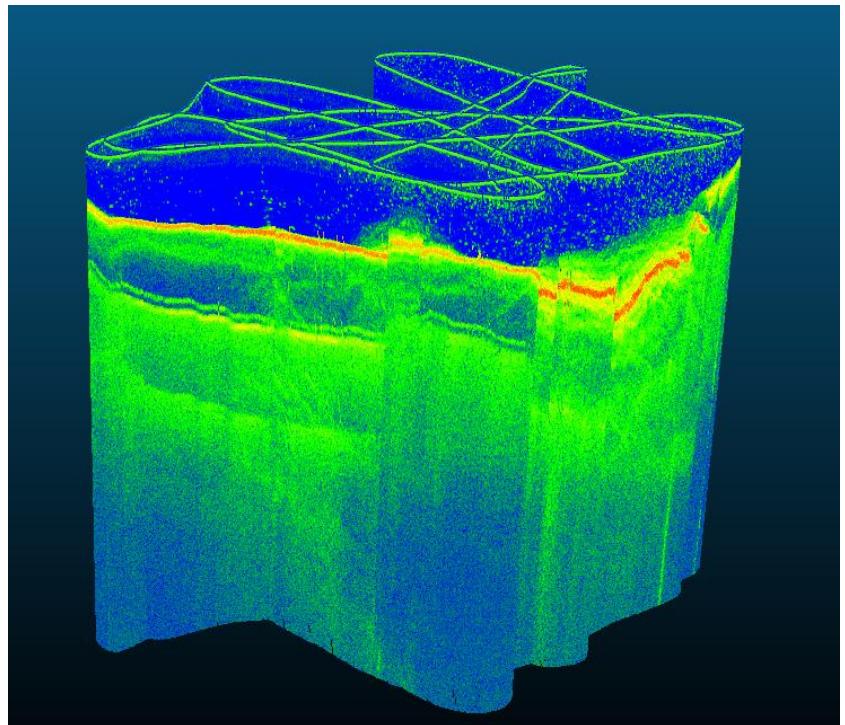


[Seaking SBP]
www.tritech.co.uk



Subbottom profiling

- Difficult to interpret data
- Echo returns very sensitive to multipath and sonar parameters
- Sonar does not identify material only reflective frontiers (that occur with changing of rock density)

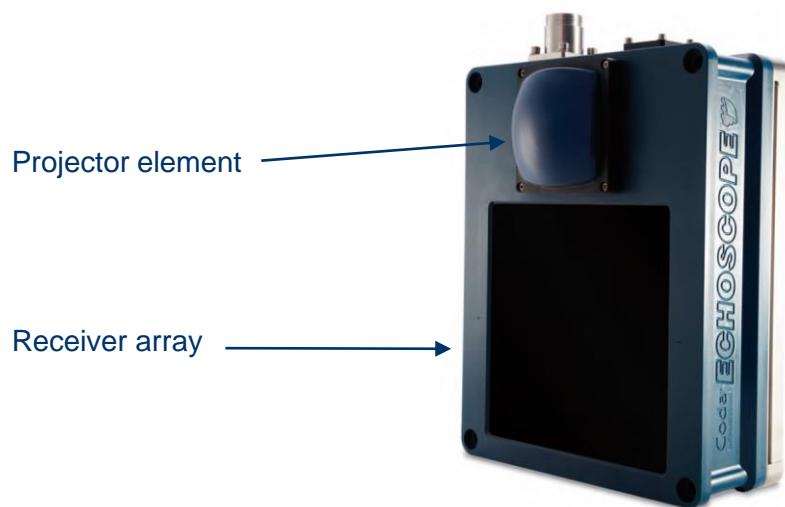
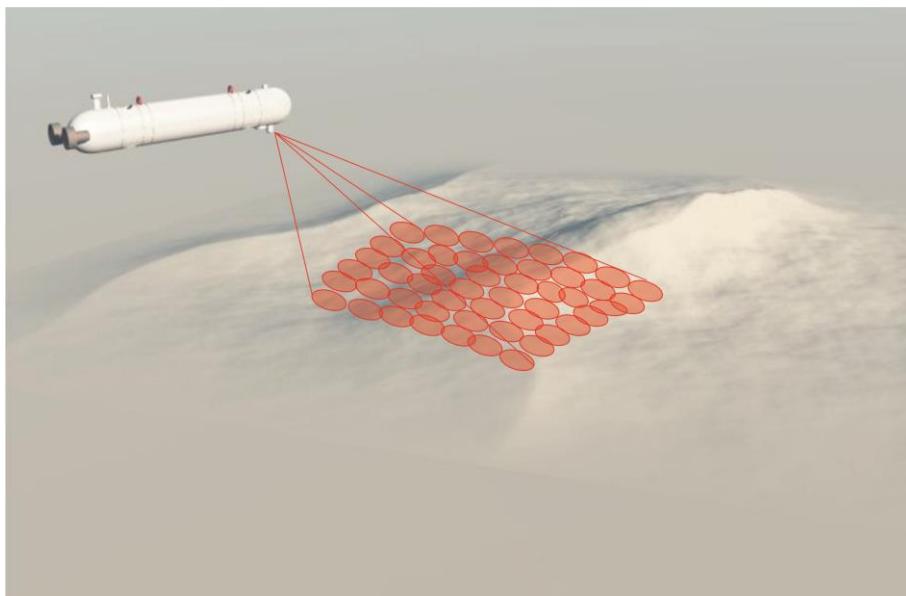


Subbottom profile of area in river performed with ROAZ autonomous surface vehicle



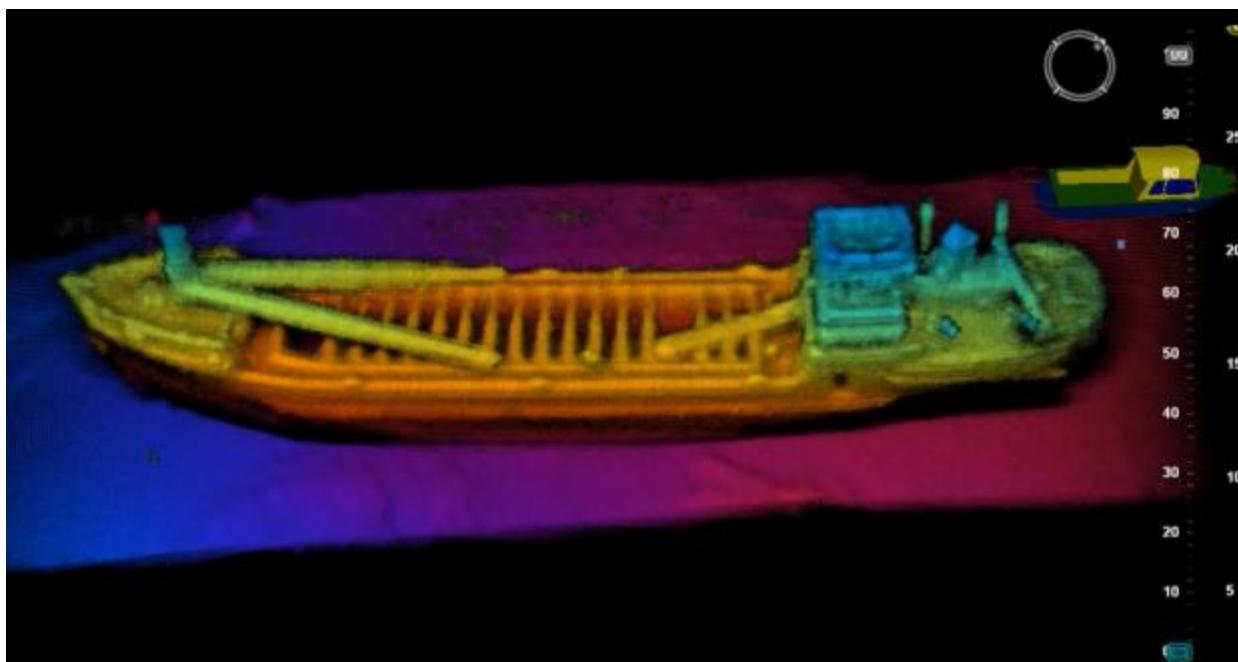
Acoustic cameras

- Projector sonar insonifies the environment
- A 2D receiver array receives echoes from multiple beams
- 2.5D snapshot (similar to a ranging ToF camera)
- Currently only one commercially available sensor: CODA Octopus Echoscope
- 375KHz / 610 KHz
- 2D multibeam
 - 2D range snapshot
 - 128x128 beams
 - 3cm range resolution
 - FAT or Max return





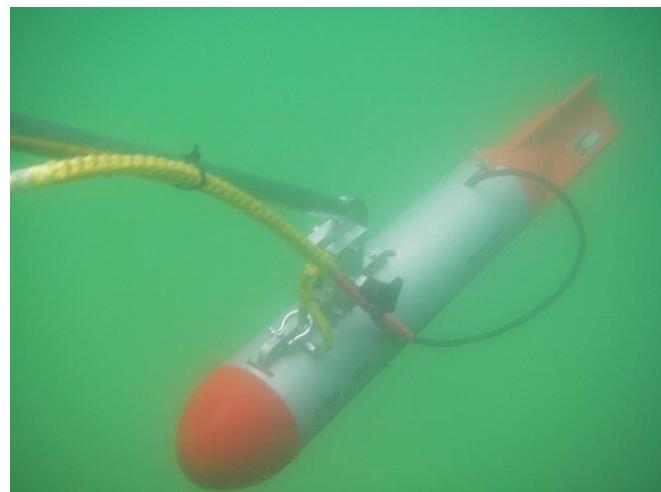
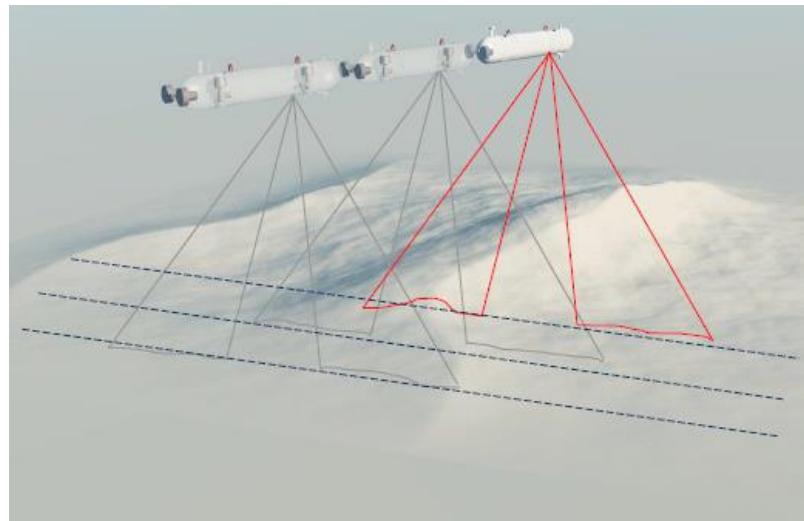
3D Acoustic cameras





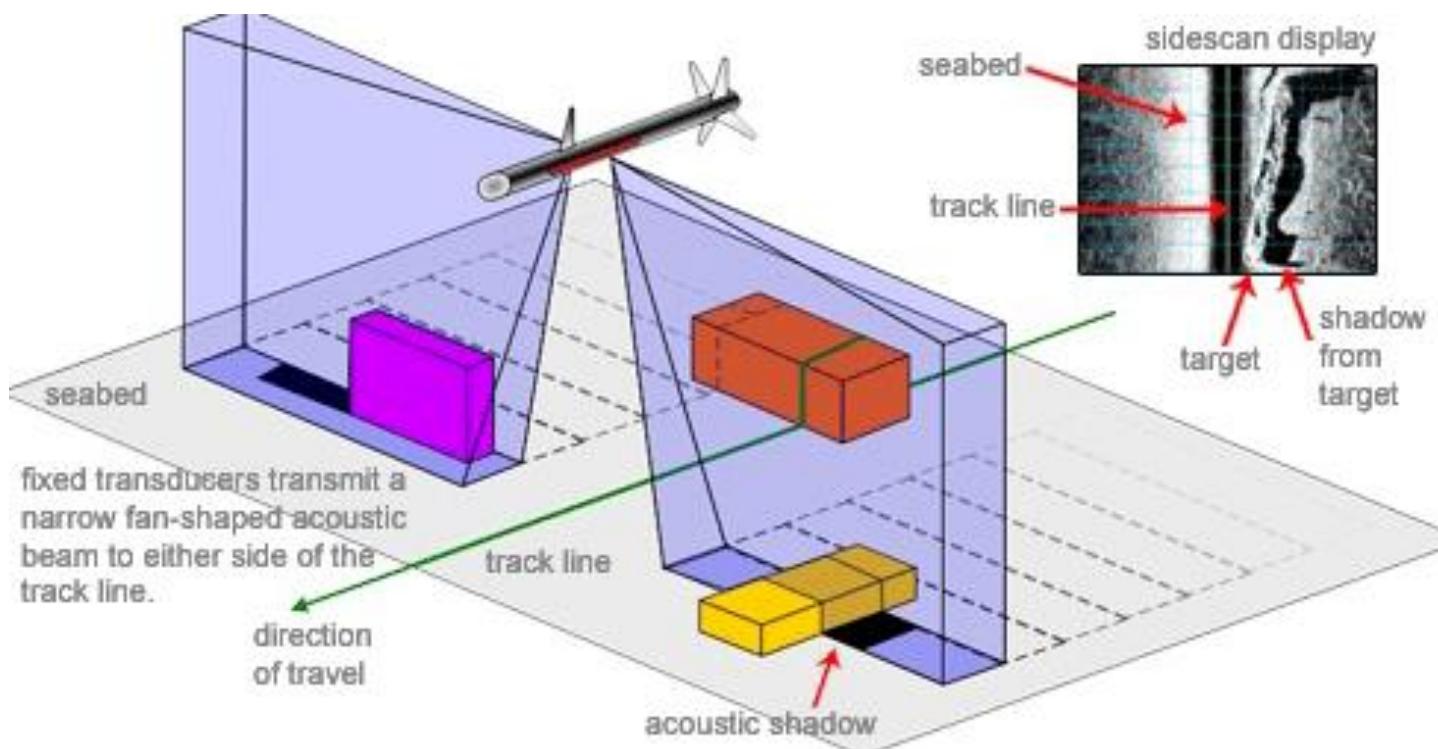
Sidescan sonar (SSS)

- Sonar “illuminates” the environment and measures signal return in two transverse narrow beams
- Most common imaging sonar
- Typical configuration with two transducers
 - Portboard side
 - Starboard side
- Does not measure signal travel time
- Measures energy of the return, the full continuous echo
- By stitching several scans, a photo-like acoustic image of seafloor can be built
- Motion is required in order to obtain an image



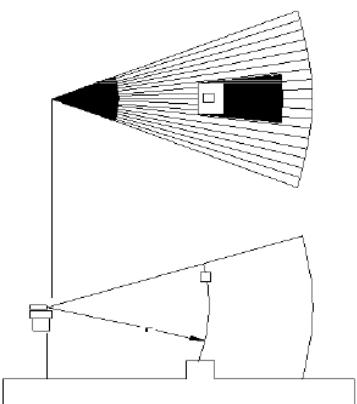


Sidescan Sonar

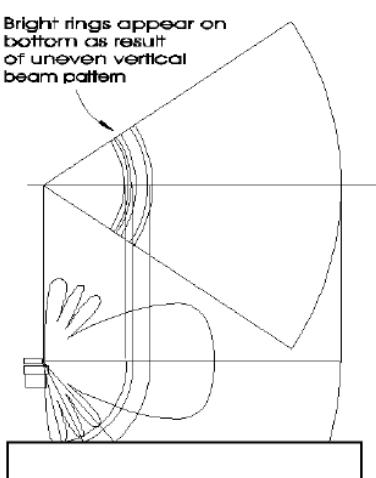




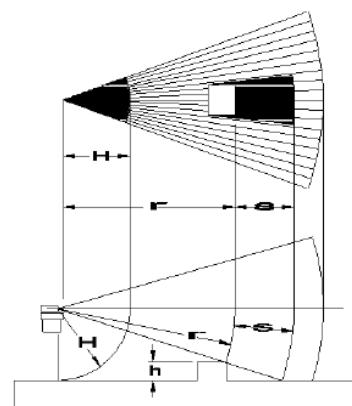
Sidescan Sonar



TARGETS AT THE SAME SLANT RANGE
BUT DIFFERENT ELEVATIONS PLOT AT THE
SAME LOCATION ON THE DISPLAY



ACTUAL SONAR BEAM WITH
UNEVEN PATTERN



$$\text{Target Height } h = \frac{H \times s}{r + s}$$

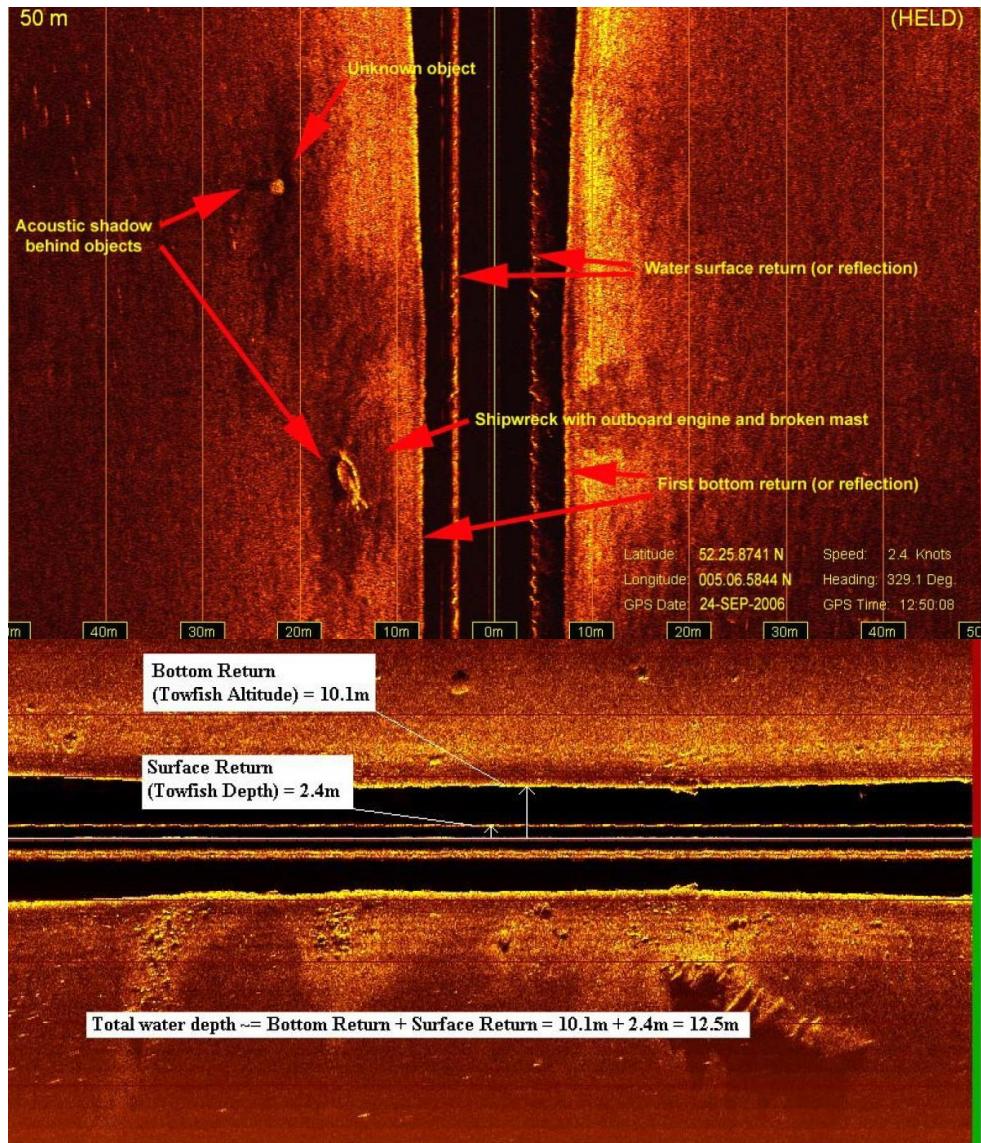
(true only on flat, level bottom)

USE SHADOW LENGTH
TO CALCULATE TARGET HEIGHT

Images from Imagenex
www.imagenex.com

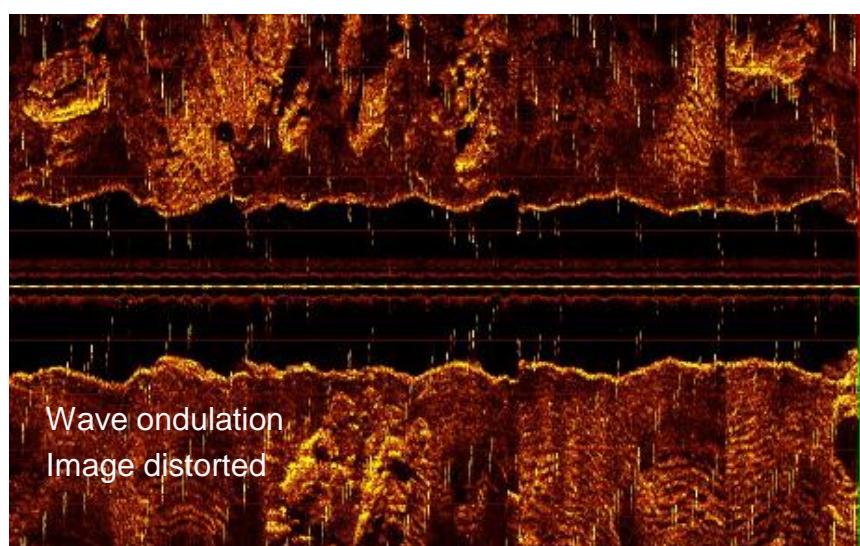
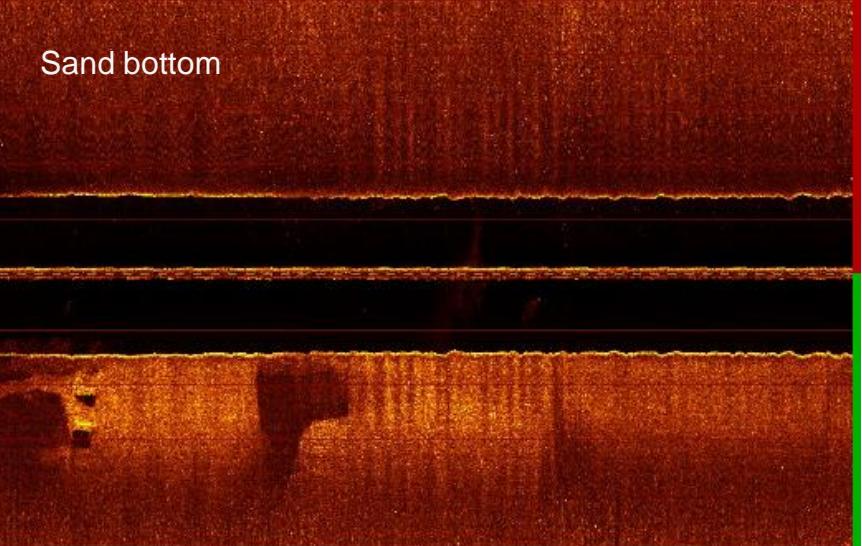
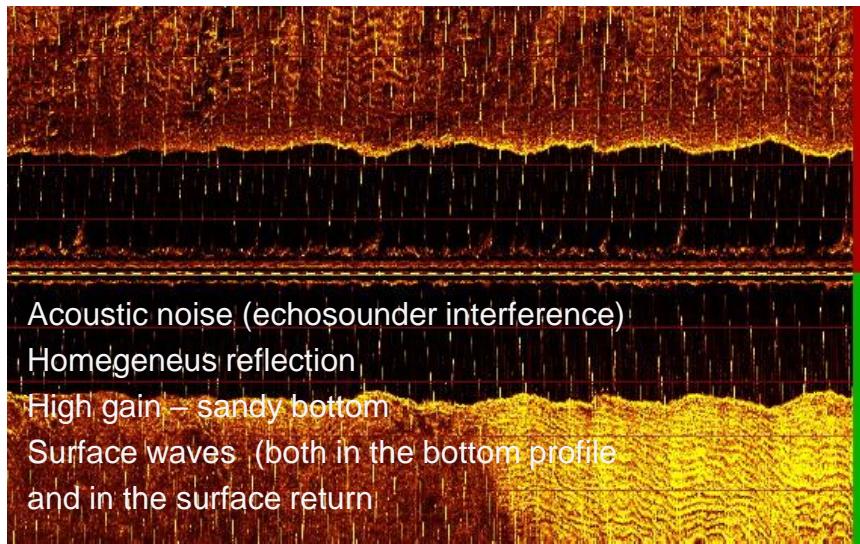
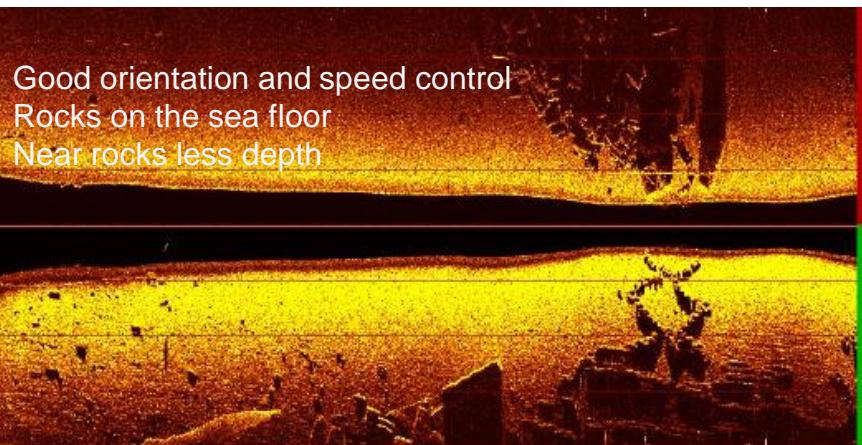
SSS image interpretation

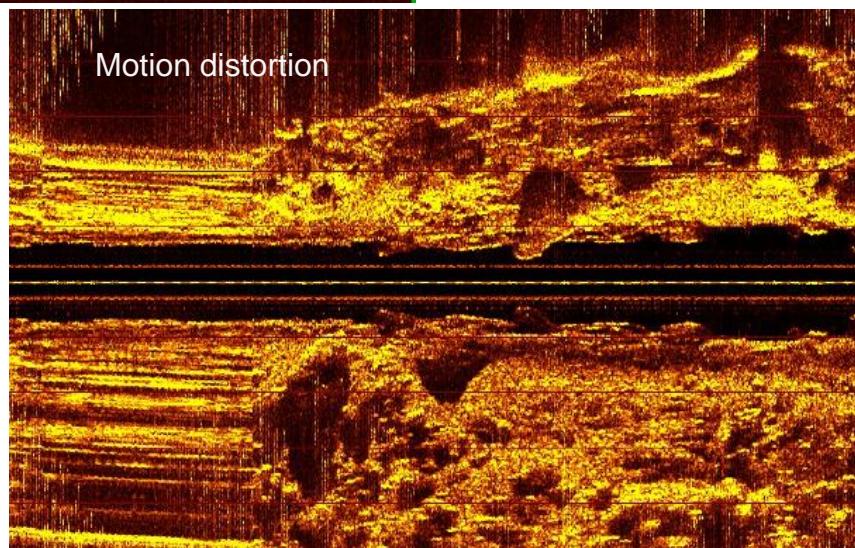
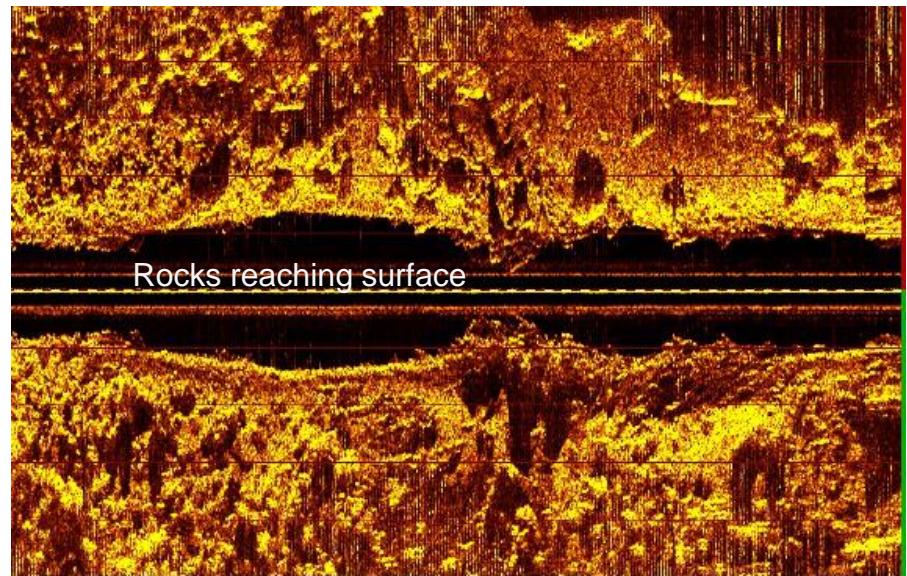
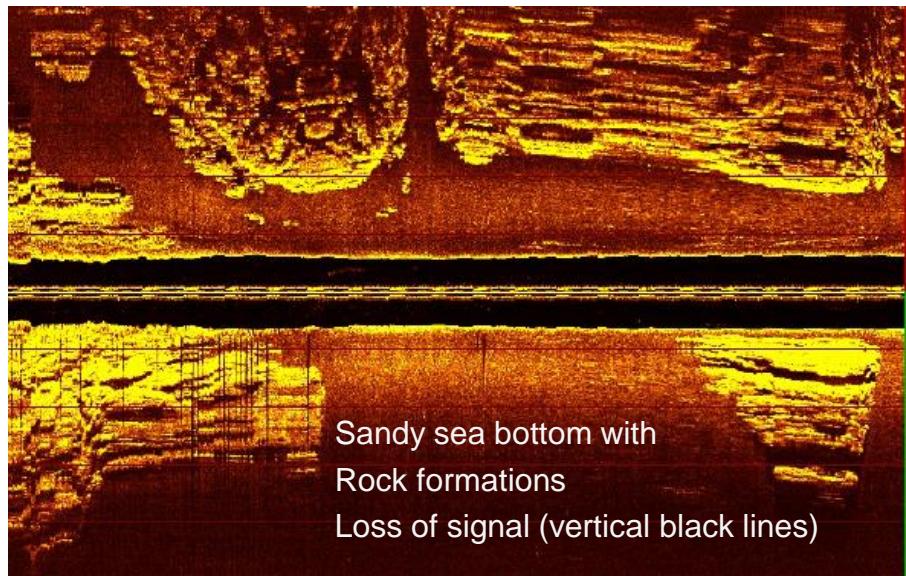
- Bottom reflectivity appears as bright spot
- Sensitivity to motion (orientation, disturbances)



Images from **IMAGENEX**

SSS image interpretation

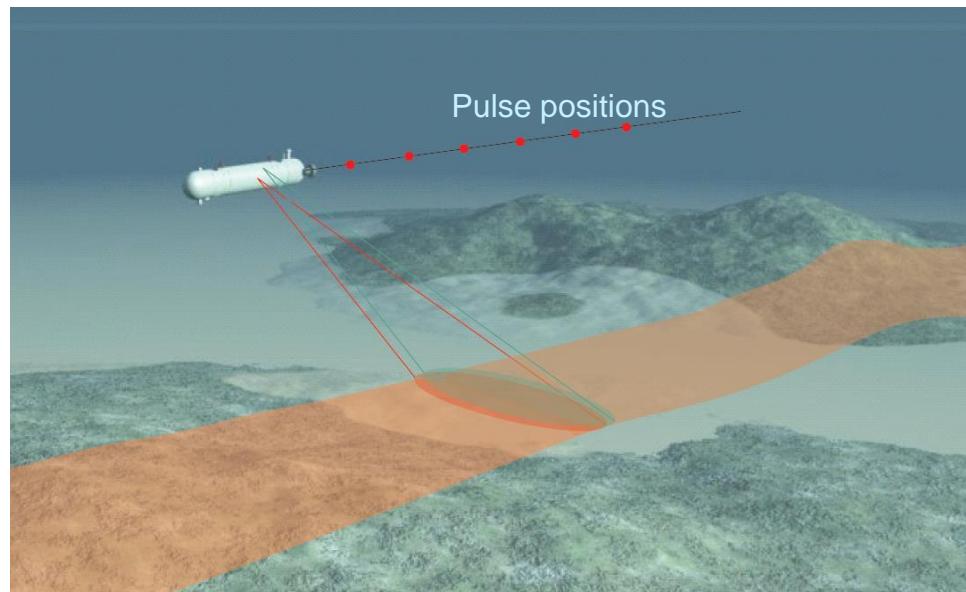






Synthetic aperture sonar - SAS

- Uses vehicle motion to “replicate” effect of having a very large transducer array (larger than the vehicle itself)
- Combines successive pings at known positions along track in order to provide a virtual larger array
- High resolution with relatively low frequency, thus also higher ranges
- Expensive
- Relatively large
 - Kraken Aquapix
 - Kongsberg HISAS



Interferometric SAS

- Combined bathymetric scan with imaging
 - Interferometric
 - Uses time difference of arrival from bottom echo with 2 vertical separated transducers
-

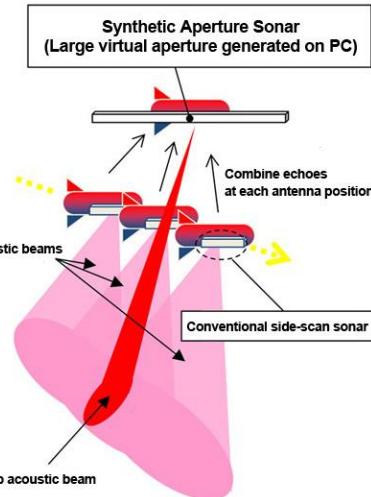
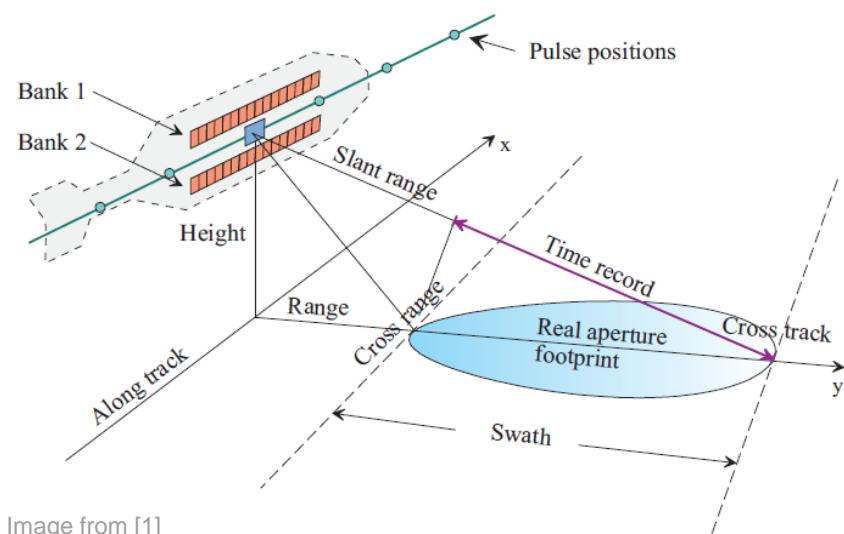


Image from
[http://www.jams.tec.go.jp/e/abou
t/press_release/
20090806/](http://www.jams.tec.go.jp/e/about/press_release/20090806/)



SAS Sonar

- SAS dependent of quality of navigation
- Relative pulse positions must be known with precision
- DPCA / Displaced Phase Centre Antenna
 - Phase measurements to provide inter-ping navigation

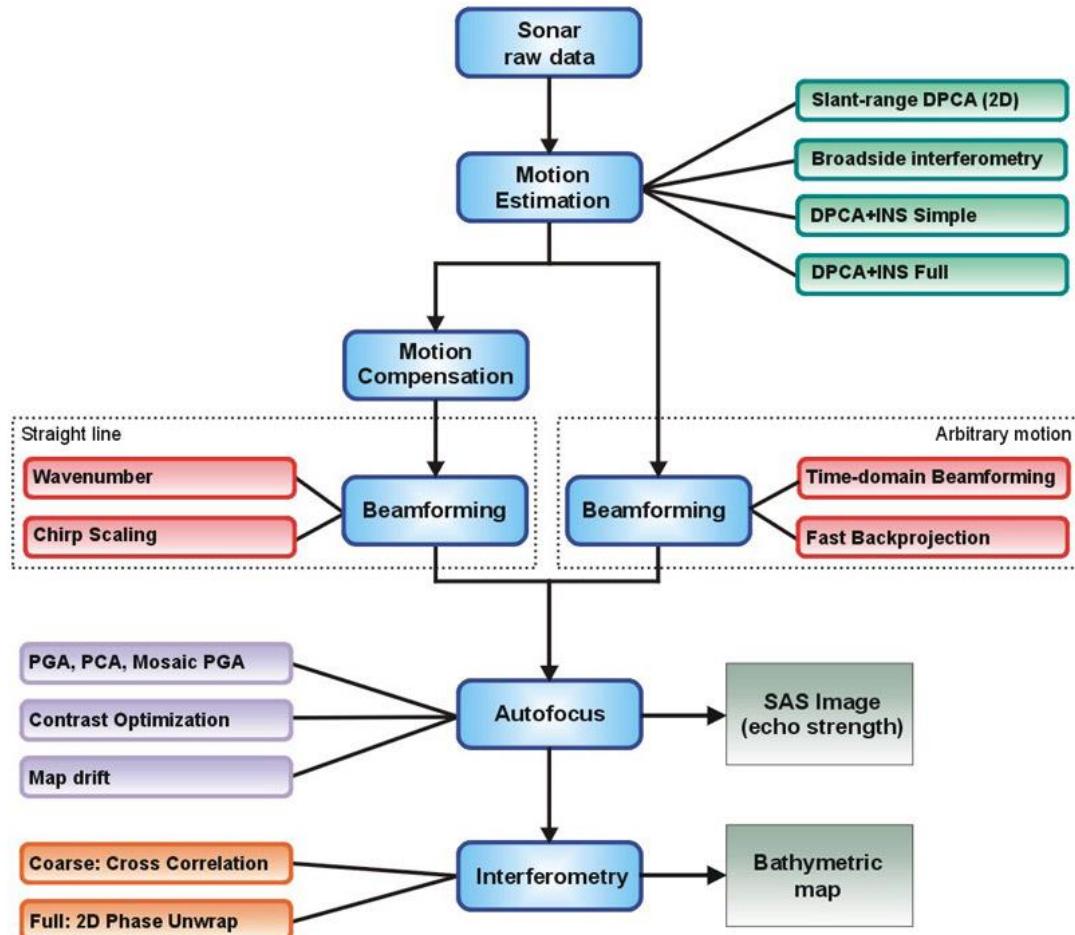


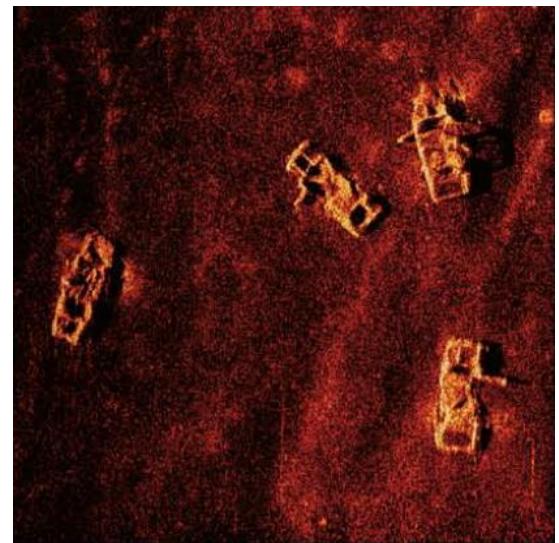
Image from [1]



Synthetic Aperture Sonar - SAS

- High resolution
- HiSAS 1030
 - Freq. 50-120kHz
 - Along track, Across track resolution
 - Max range @ 2m/s (swath 400m)
 - Area coverage rate 2km²/h

Images from Kraken
AquaPix Datasheet
www.krakernsonar.com





Doppler Sonars

- Use Doppler frequency shift to measure relative velocities
 - Frequency shift of a wave for observer moving relative to its source
 - Higher frequency on approach and lower in separation
- Sonar emits at frequency and detects frequency shift, thus measuring velocity towards target
- Target can be either the sea-bottom or other infrastructure or particles in suspension on water (ADCP)



DVL – Doppler Velocity Log

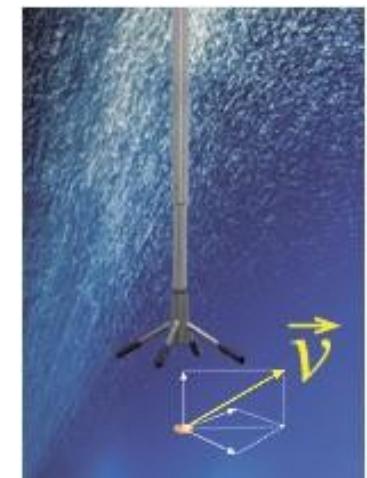
- With bottom tracking for navigation
- 2 types :
 - Multiple beam (4 beams or more)
 - Phased array (beam forming)
- Multiple beams allow for determination of the 3 components of the instrument velocity (the doppler effect only provides relative velocity in de direction of each beam)
- Available OEM versions for AUV integration (transducers separated from electronics)
- Dual head options
- Return also altitude and one water cell velocity
- DVL (bottom tracking, <100 m, typically) + IMU
 - Accuracy 0.1-0.5% travelled distance





ADV -Acoustic single cell water velocimeter

- Returns water velocity in a small single cell
- In robotic vehicles used for water relative velocity measurement
- Turbulence measurements
- High precision
- High measurement rate

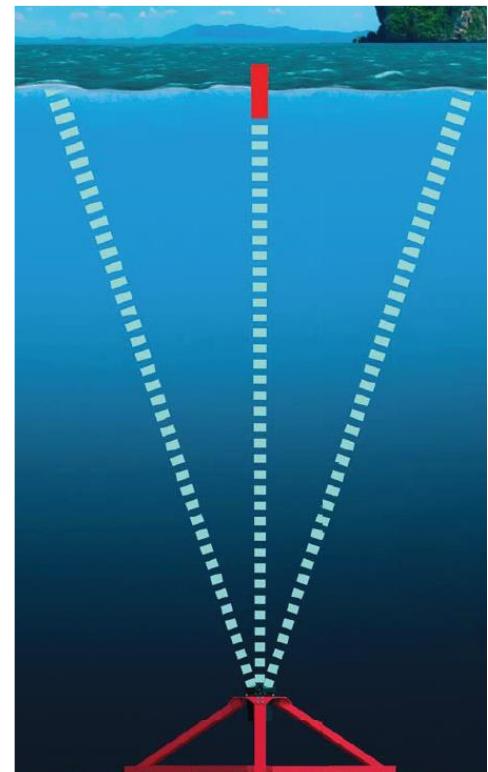


www.nortek-as.com



ADCP – Acoustic Doppler Current Profiler

- DVL with return of velocities of multiple cells in the water column (water current profile)
- Can be an option in navigation DVL instruments
- Usually mounted on the sea floor pointed upwards
- Surface tracking allows for wave measurement and surface current



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Imaging





Cameras

- Provide high resolution images
- Robust landmark detection can be performed
- Light quality degrades underwater
 - Chromatic distortion
 - High sensitivity to turbulence and backscatter



a)



b)



c)

a) diffusion b) flickering c) non-uniform lighting

Stereo Vision

- Stereo vision underwater has limited applications
- Old experiments underwater
 - Jean de Wouters d'Oplinter, 1948 with tests in Mediterranean sea
 - Rebikoff, 1954 used stereo for mapping archeological sites on manned vehicle Pegasus
- Standard application of stereo techniques



Hyperspectral cameras

- Acquire data along the electromagnetic spectrum, from ultraviolet to long-infrared
- When all data is associated, it can be generated a hyperspectral image cube, which consists of a set of images layered on top of one another.
- Used off-water for material analysis and identification (ex: agriculture, mining)

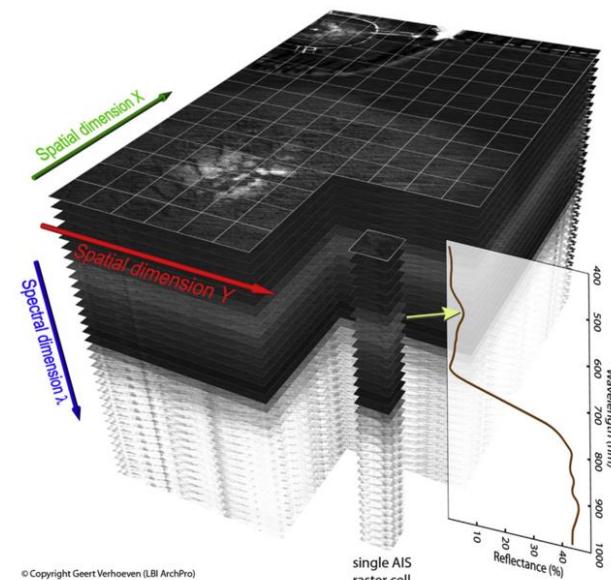


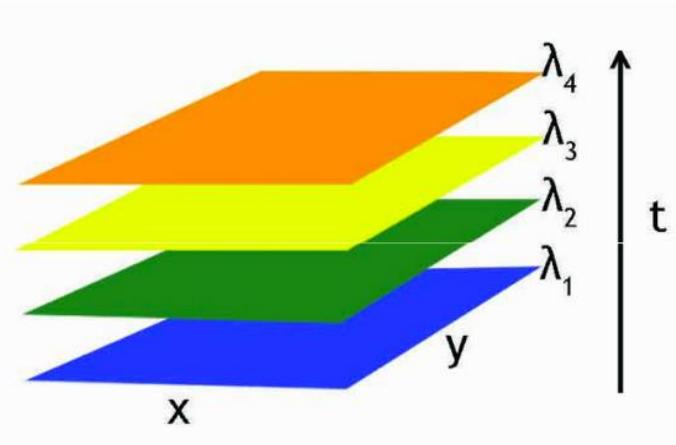
Image from [1]

[1] Michael Doneus, Geert Verhoeven, Clement Atzberger, Michael Wess, Michal Rus, "New ways to extract archaeological information from hyperspectral pixels"



Hyperspectral Cameras – Tunable Spectral Filter

- Acquire 2D image at a time, wherein each 2D image corresponds to a wavelength
- So, only when all wavelength (2D image) have been acquired, it becomes possible to obtain the spectrum of each pixel.



[1] <http://www.gildenphotonics.com/hyperspectral-imaging-/hyperspectral-imaging-technology.aspx> .

Hyperspectral Cameras - Pushbroom

- Allows full spectral data simultaneously, with spatial line scanning over time
- The camera acquires all spectral information exactly at the same time, being insensitive to instrument/sample movement
- For each line, obtains the spectrum for each pixel

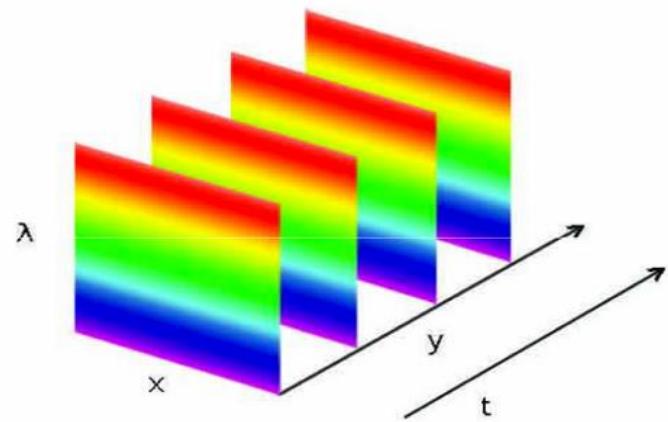


Image from [1]

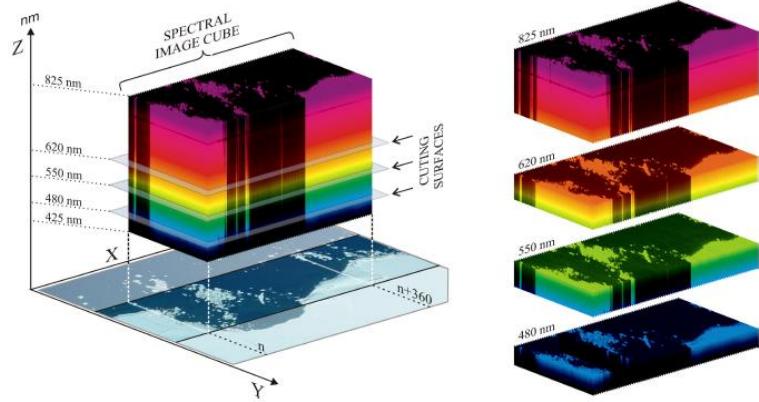


Image from [2]

[1] <http://www.gildenphotonics.com/hyperspectral-imaging-hyperspectral-imaging-technology.aspx> .

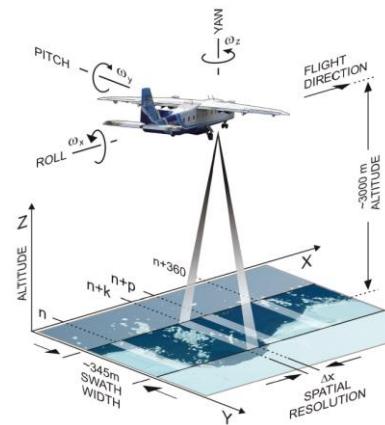
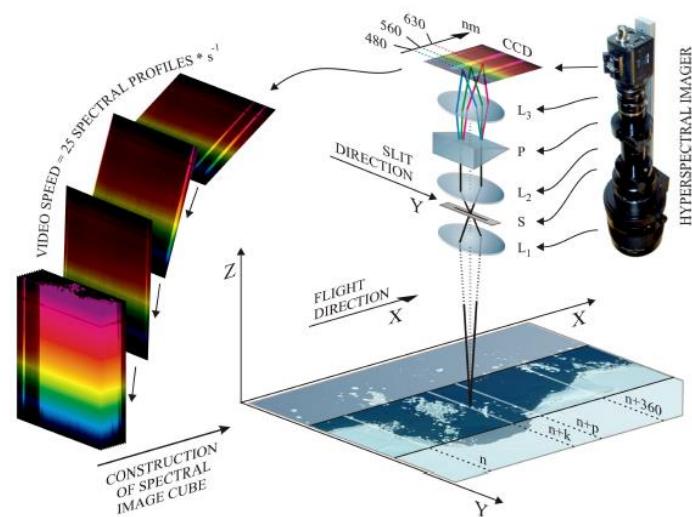
[2] Volent, Z., et al. "Kelp forest mapping by use of airborne hyperspectral imager." *Journal of Applied Remote Sensing* 1(1), 2007.

Unmanned Autonomous Vehicles in Air, Land and Sea | Polit. Milano 2016



Underwater Hyperspectral

- Pushbroom camera
- Airborne sensor (not really underwater!!!!)
- Similar approach as system used in Sunny project
- Kelp algae studies
- Underwater hyperspectral system under development for mining studies
- Some commercial developments (ex: Ecotone UHI)



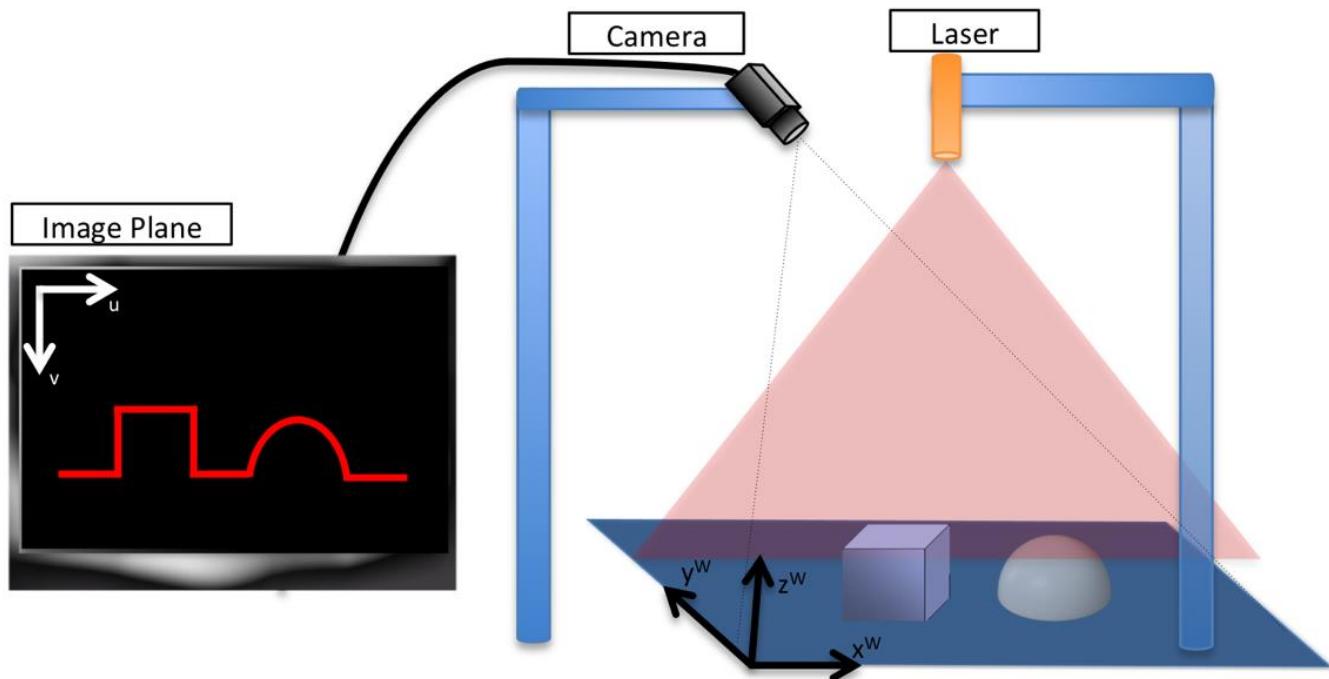
Images from [1]

[1] Volent, Z., et al. "Kelp forest mapping by use of airborne hyperspectral imager." Journal of Applied Remote Sensing 1(1), 2007.



Structured light

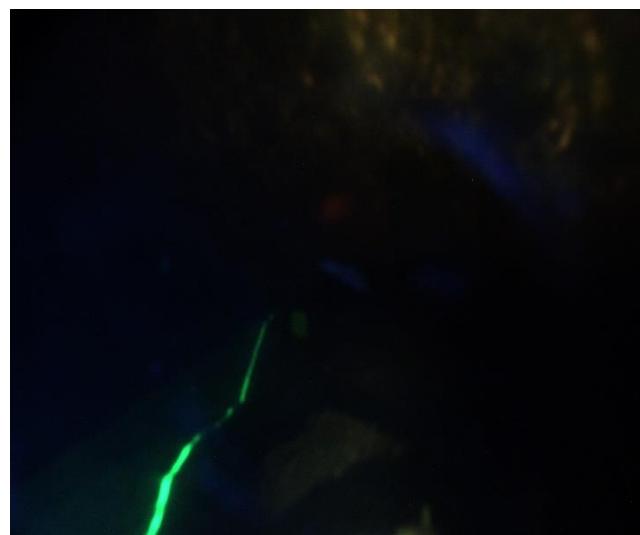
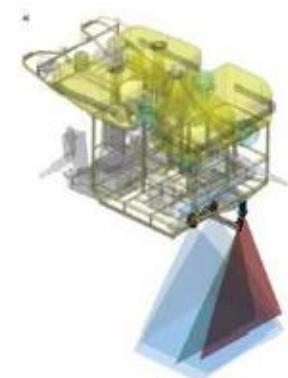
- The use of pure visual perception methods is limited in underwater environments.
- One way of overcoming such limitation is by using line laser projector.





Structured light

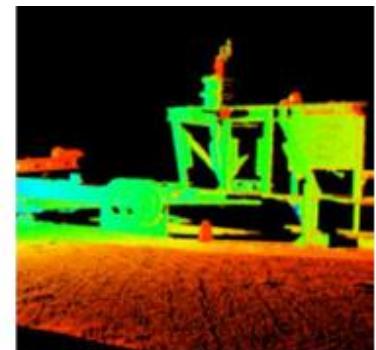
- Current high degree of application in microbathymetry and close range profiling
- Common use of simple dual dot laser projectors in teleoperated ROV missions to provide scale in imaging for the user
- Blue and green laser
- Very sensitive to water turbidity
- Excellent precision when comparing with sonar based sensors
- Comercially available solutions





Underwater LIDAR

- Standard LIDAR in water environment
- Distance measurement by time of flight
- Technology in water still in its infancy
- Commercial solutions have been proposed (3D at Depth)
- Suffers from the same limitations from structured light and more expensive





Commercial Underwater Laser Systems

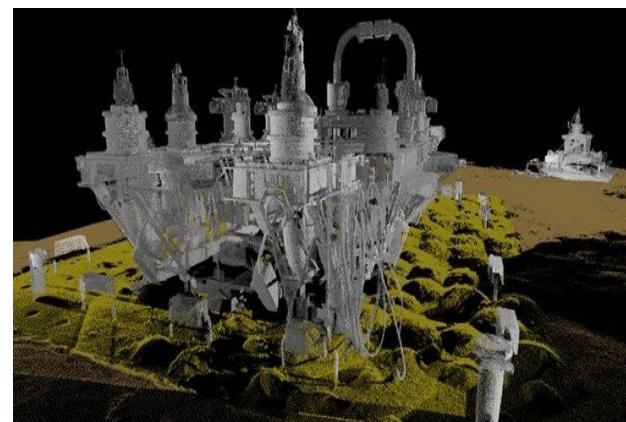
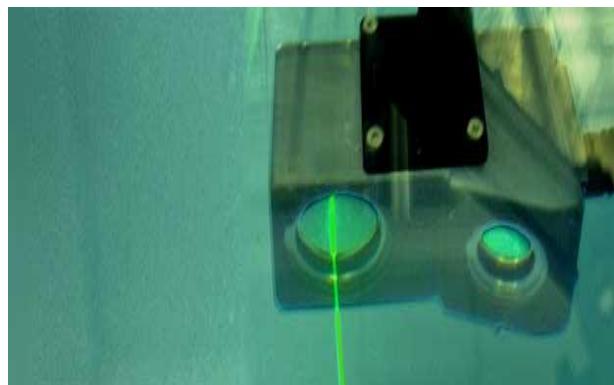
- 3D at Depth (lidar)



Seatronics Ag2R



Newton Labs M210UW



https://youtu.be/kjSuwofgt_g

Navigation



Navigation problem

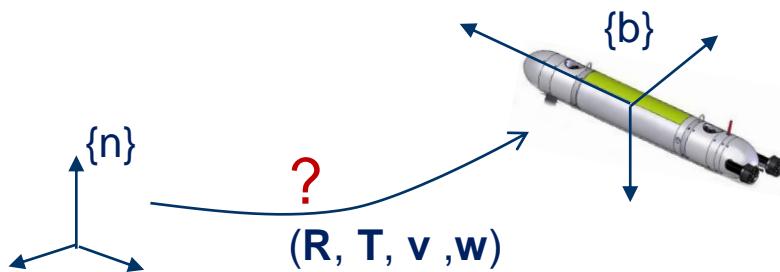
Where am i?

- In relation to a pre-defined reference frame
 - Inertial frame or fixed in the world (NED, ECEF)
 - Local reference frame
- Relative localization
 - In relation to temporary frame (ex: landmark)
 - In relation to mobile reference frame
- Topology
 - Order relation with environment elements
- Semantic

Navigation problem

Where am i?

- Required information depends on application
 - Position and orientation (relation between $\{b\}$ and $\{n\}$)
 - Velocity information (linear, angular)
 - Qualifiers (ex: on the right of, inside etc)
 - Deterministic or probabilistic (more useful, in this case some measure of uncertainty is required)



Estimate position, orientation and velocity of the vehicle



Localization methods

- Dead reckoning – relative measurements
 - Inertial navigation
 - DVL
- Absolute measurements
 - Active beacons (VLS, GPS, LBL, ...)
 - Landmark recognition
- Combination of both relative and absolute measurements

Some problems in navigation...

- Errors in sensors (noise, lack of precision, accuracy, drift, bias...)
- Errors in robot model
- Errors in world model (map,)
- Difficult to obtain “exact location”
- Even approximate location (even with large errors) can be useful
- Useful to have measures on the confidence of determined information



Deterministic models do not deal with uncertainty, thus:

Probabilistic models



Probabilistic approach

- Information represented in a probabilistic way
- Takes into account uncertainty in the sensor measurements, model of environment and of the robot
- Allows to integrate in a coherent way information from different sensors
- Representation more suitable to reality

Probabilistic approach

- Information represented by probability distributions
 - PDF Probability Density Functions (continuous, discretization's ...)
 - Significative samples...
 - Statistical moments (mean, covariance, ...)
- Random variables instead of deterministic
- Robot and sensor models incorporate noise characterization measures
- Integration and probabilistic calculus

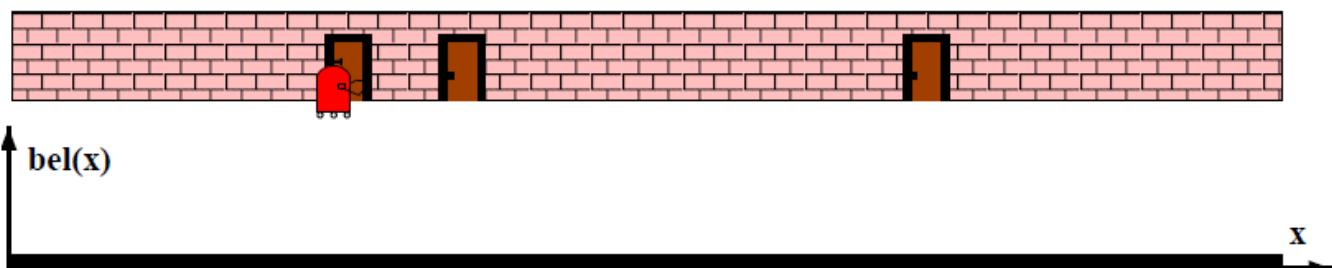


Example^[1]

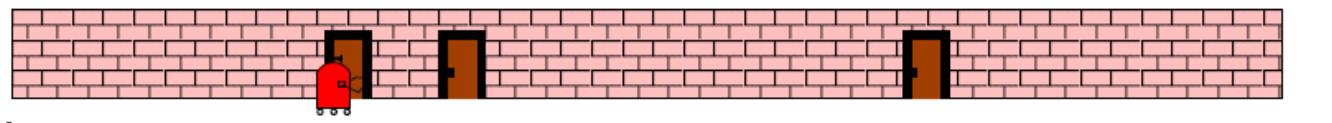
$bel(x)$ – “belief” PDF in all possible locations, best estimate or robot location in a given moment

$p(z|x)$ – conditional probability of z (measurements) given x (state)

initial estimate



a door is observed



Conditional probability of observing a door is maximum when in front of one of them



A priori I can be any one of the 3, $bel(x)$ represents this 3 possibilities



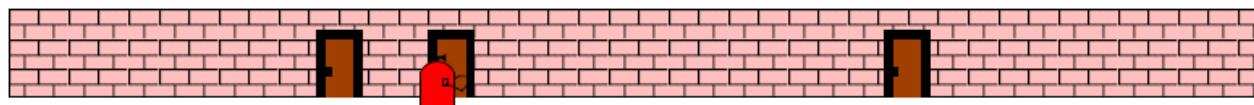
Figure from [1]

[1] S. Thrun et al, “Probabilistic Robotics”, MIT Press,



Example

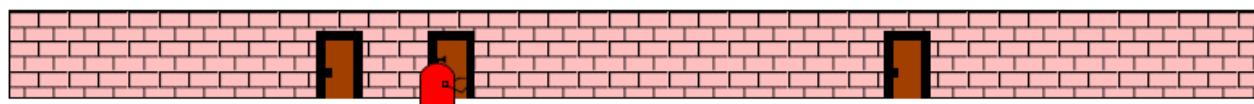
robot moves to the right



Location estimation is
“attenuated” (uncertainty
increase)

Prediction

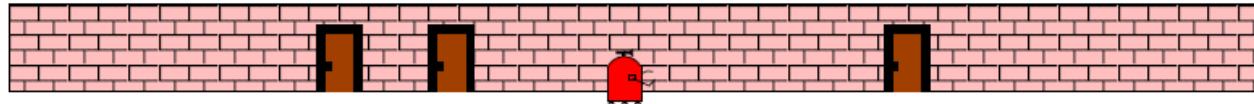
another door is observed



After observing 2nd door
incorporates information –
bel(x) has a large peak in
2nd door – consistent with
motion performed and
observation

Correction

robot moves to the right



Location estimation
degrades again

Prediction

Predict – Update cycle

- Allows to accommodate multiple hypotheses in conflict
- **Predict- Correct** cycle incorporates increases of uncertainty due to motion or passing of time as well as additional information given by observations



Markov process— Current process state contains all information required for future prediction (not needed to know all the precious history of the state)



Navigation in marine autonomous systems

- Surface systems
 - GNSS
 - INS
- Underwater robots navigation
 - Absolute acoustic positioning systems
 - SBL, LBL
 - USBL
 - Dead reckoning and sensor fusion solutions
 - DVL
 - INS
 - Terrain based navigation and SLAM



Surface systems

- GNSS – Global Navigation Satellite Systems (GPS, Glonass, Compass, Galileo) usually available and main localization mechanism for USVs
- Loran (Loran-C) – US original, low frequency radio beacon based navigation (trilateration), now almost substituted by GNSS systems
- GNSS + INS for 6DOF and additional precision
 - Many applications require precise orientation to be known (even when roll and pitch is not controlled), ex: bathymetry, visual target estimation.



GNSS – Global Navigation Satellite Systems

- Geo-spatial positioning using time signals transmitted by satellites
- High precision synchronized satellite clocks allow for receiver to determine its position
- Standard regime, receiver needs at least 4 satellites (3 for position and one extra for the drift in receiver clock)
- Signal degradation due to poor geometry, reflection and multipath
- Multiple constellations
 - GPS – USA
 - GLONASS – Russia
 - Galileu – Europe
 - BeiDou - China

Global Positioning System (GPS)

- Enables three-dimensional positioning and time synchronization to UTC time
- Civil GPS receivers can generate 3 types of measurements
 - Pseudorange (PR) measurements
 - Phase Measurements
 - Doppler Measurements



GPS-Pseudorange

- Each satellite transmits a unique ranging signal with embedded time information, C/A code
- By decoding the signal, receivers determine the time of transmission from the satellite
- Ideally (clocks synchronized), travel time multiplied by the speed of light provides the receiver/satellite range.
- In reality, clock biases exist, hence the word pseudorange

Considering only clock biases, pseudoranges are given by:

$$PR = d + c\delta t_S - c\delta t_R$$

$d(m)$ - True satellite/receiver range

$c\delta t_S$ (s) - Satellite clock bias relative to GPS time

$c\delta t_R$ (s) - Receiver clock bias relative to GPS time

$c(m/s)$ - Speed of light

GPS- Pseudorange

- Other sources of error can exist

$$PR = d + c\delta t_S - c\delta t_R + \delta d_{orb} + \delta d_{iono} + \delta d_{tropo} + \delta d_{multi} + \eta$$

$\delta d_{orb}(m)$ - Orbital error

$\delta d_{iono}(m)$ - Ionospheric error

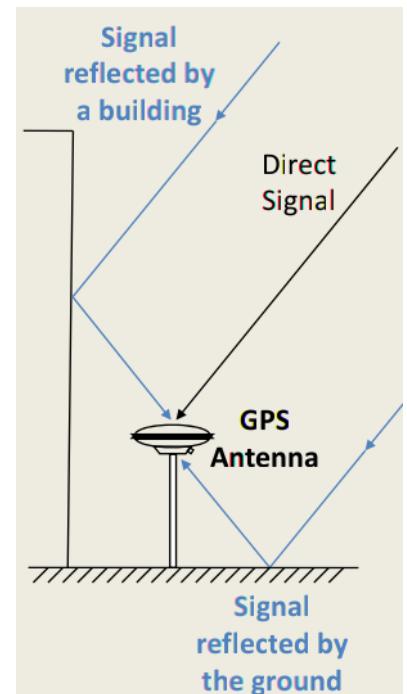
$\delta d_{tropo}(m)$ - Tropospheric error

δ_{multi} - Multipath error

$\eta(m)$ - Receiver noise

- Multipath can be a problem in the water surface
 - Typical remedies are to shield the reflections in the base of antenna
- In single point positioning expected accuracy around 10m

Figure from [1]



[1] "GNSS Precise Positioning with RTKLIB", 2011

[2] Antonio Angrisano, "GNSS/INS Integration Methods", 2010

GPS – Phase Measurement

- Tracks the phase of the carrier frequencies (L1 or L2)
- Phase measurement multiplied by the carrier wavelength represents the satellite/receiver range.
- Satellite/receiver range is expressed in units of cycles of the carrier frequency
- Exists an ambiguity term representing the number of wavelength cycles (determined with help of a base station)
- Millimeter accuracy measurement

$$\lambda\phi = \lambda N + d + c\delta t_S - c\delta t_R + \delta d_{orb} + \delta d_{iono} + \delta d_{tropo} + \delta d_{multi} + \eta$$

λ - Carrier wavelength

ϕ - Phase measurement

N - Number of cycles in the satellite/receiver distance



GPS – Doppler Measurement

- Derivative of the carrier phase
- Represents the frequency shift caused by the relative receiver-satellite motion
- Multiplied by the carrier wavelength gives the derivative of the satellite/receiver range, used to compute the receiver velocity

GPS Processing Techniques

- Differential GPS

- Static receiver – Base station
- Compared measurements to the moving receiver (Rover) to minimize system errors
 - Ionospheric and tropospheric delay
 - Clock Bias
 - Ephemeris errors
- Can be used online or in post-processing



Image from]“GNSS Precise Positioning with RTKLIB”, 2011

GPS Processing Techniques

- Real- Time Kinematic (RTK) GPS
 - Uses phase measurements
 - Base station helps to solve wavelength cycle ambiguity
 - Reduces errors:
 - Atmospheric delays
 - Internal receiver errors
 - Attenuates multipath errors
 - RTK requires 5 visible satellites
 - Millimeter accuracy



Inertial Navigation Systems

- Dead reckoning navigation systems
- Inertial sensors
 - 3 Accelerometers estimate linear motion
 - 3 Gyroscopes estimate angular motion
- Unbounded position error growth
- Multiple levels of sensor integration
 - IMU raw sensor outputs (accels. and angular velocities)
 - AHRS – Attitude Heading and Reference Systems (provide integration to provide heading or vertical direction)
 - Full navigation solutions – INS
 - Integrated GNSS-INS or (as in marine applications possible integration of DVL data, ex: IxSea Phins)

INS working principles

- Initial INS systems used gyro-stabilized platforms
 - High grade (navigation grade or strategic) still in this technology
- Most INS systems currently are **strapdown**
 - Without moving parts
 - Developments in MEMS coupled with widespread applications (ex: cellphones) pushed development
- Accelerometers measure specific force

$$f = a - g$$

a Kinematic acceleration

g Gravitational acceleration

- Gyroscope sensors usually fall into 3 categories
 - MEMS – oscillating micro devices detecting angular motion
 - FOG – Fiber optic gyro
 - RLG – Ring laser gyro

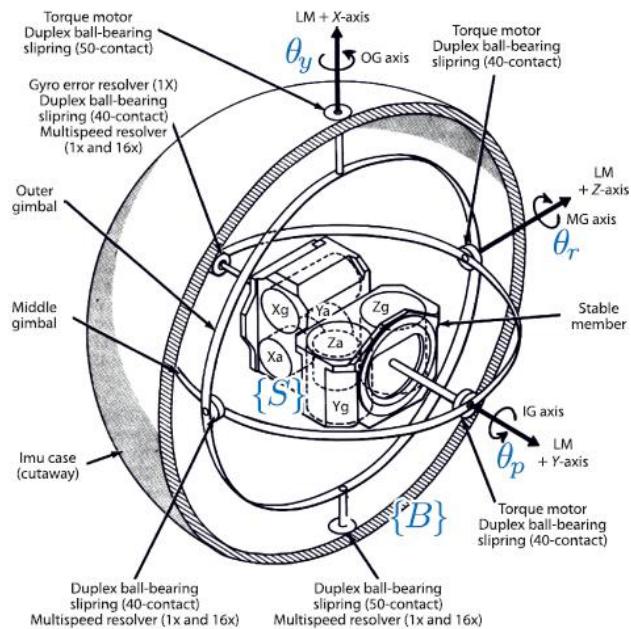


Figure from [1]

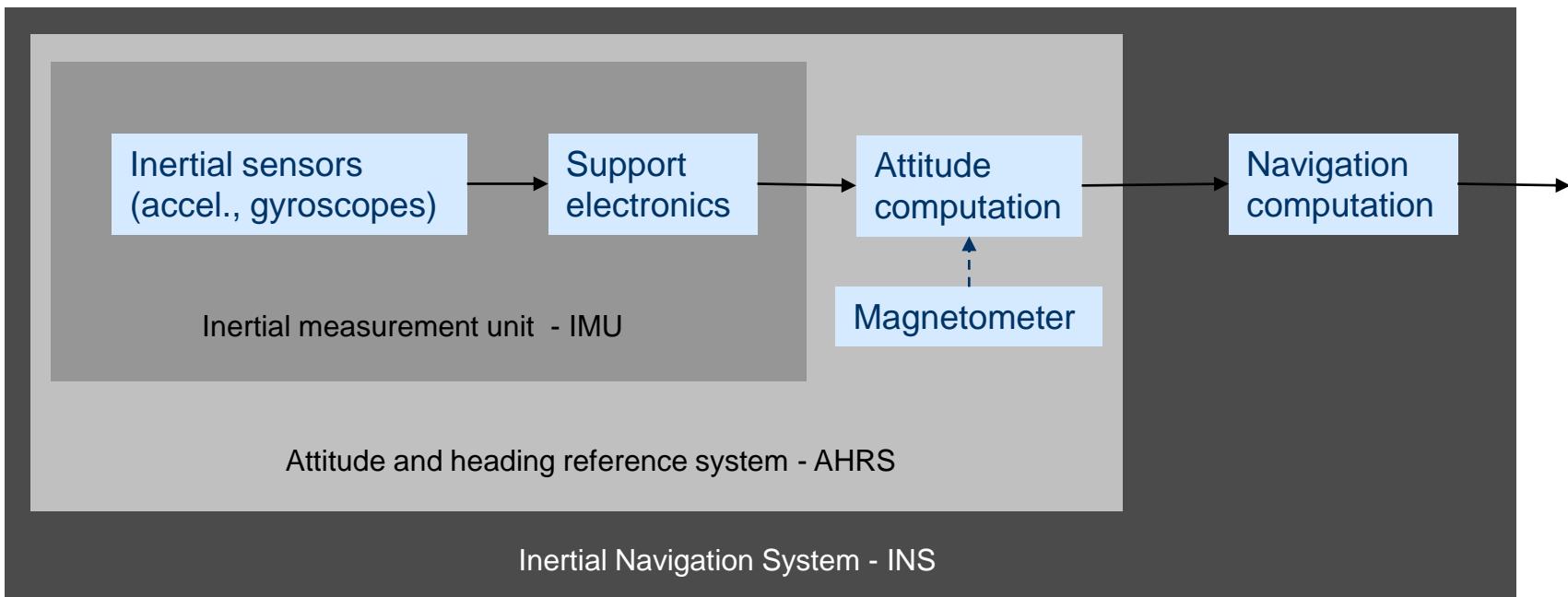


Novalabs IMU module

[1] image adapted from (Apollo Operations Handbook, LMA 790-3-LM), in P. Corke, "Robotics Vision and Control", Springer, 2011



Inertial Navigation Systems



Analog Devices
ADRXS610 gyroscope



ISEP embedded
IMU unit



Xsens Mti-10 AHRS



iMAR iNAV-FMS-E



Fiber Optic Gyros

- FOG and RLG gyros measure rotation using the Sagnac effect
 - Rotation causes the light path in a fiber optic coil or ring laser path to be longer in the direction opposite to the rotation
 - Two light signals are “sent” to the path and the phase difference measures the motion

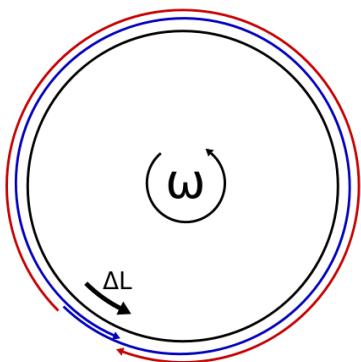


Image: “Ring laser gyroscope produced by Ukrainian “Arsenal” factory on display at MAKS-2011 airshow#, by James Nockson

INS Mechanization

- Integration of the raw accelerations and angular velocities to provide position and orientation

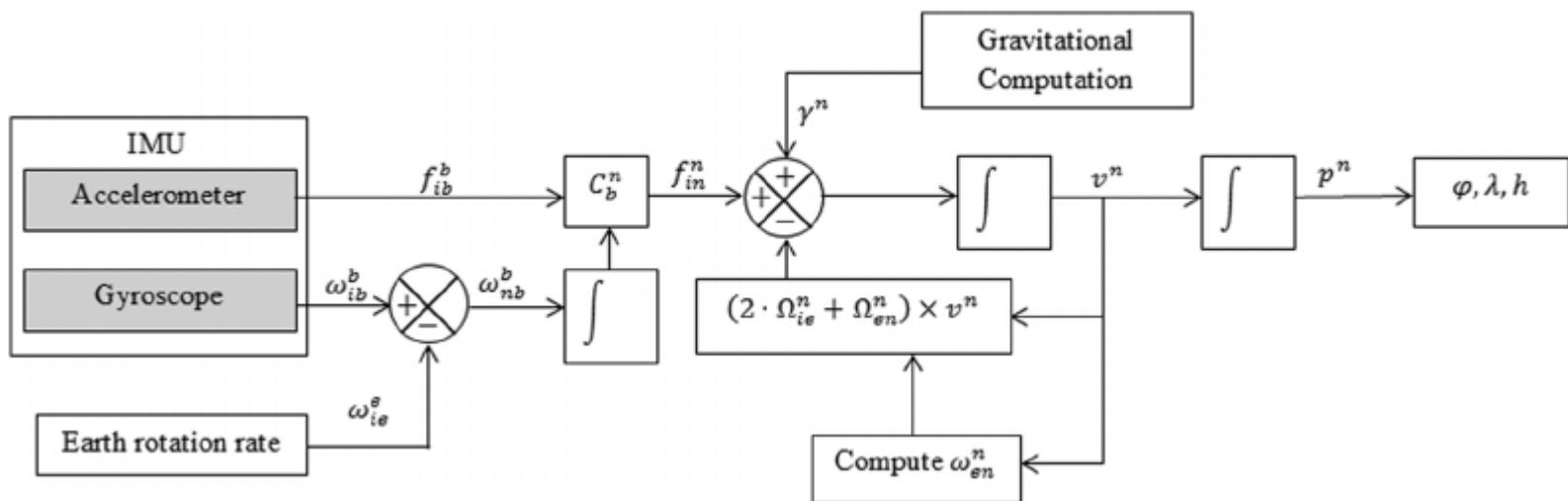


Figure from [1]

[1] Y. Yoon et al “Real-time precision pedestrian navigation solution using Inertial Navigation System and Global Positioning System”, Advances in Mechanical Engineering 7(3) · March 2015

Wide range in quality

- From consumer to strategic
 - Ex: embedded IMU with $\sim 15^{\circ}/\text{hr}$
 - Spacial FOG with $\sim 1^{\circ}/\text{hr}$
 - Phins $\sim 0.1^{\circ}/\text{hr}$ (0.6 naut miles/hr)
- North seeking capabilities
 - Detecting earth rotation and determine Earth spinning axis direction (north)
 - Degrades with latitude

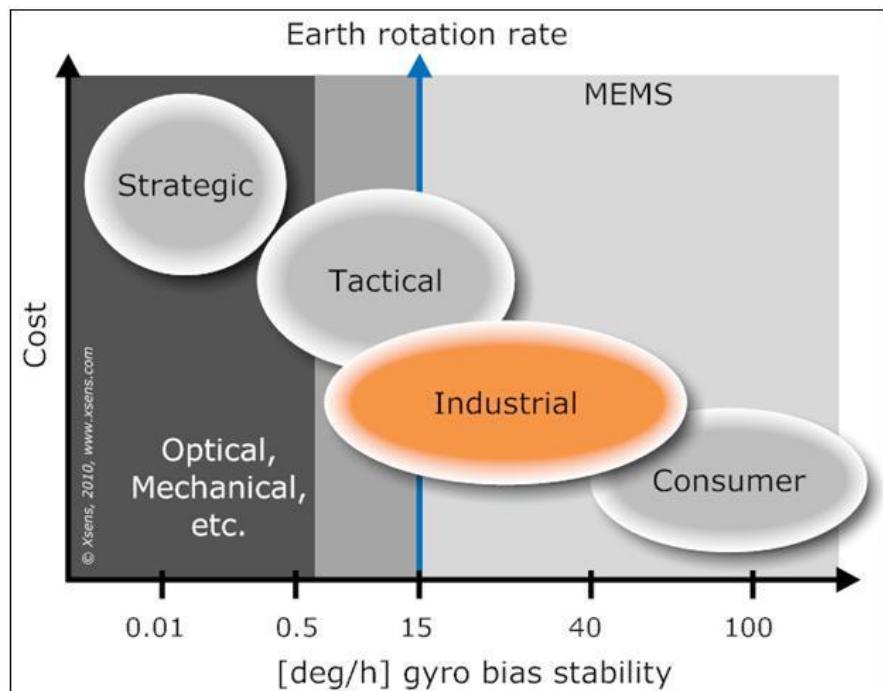


Figure from [1]

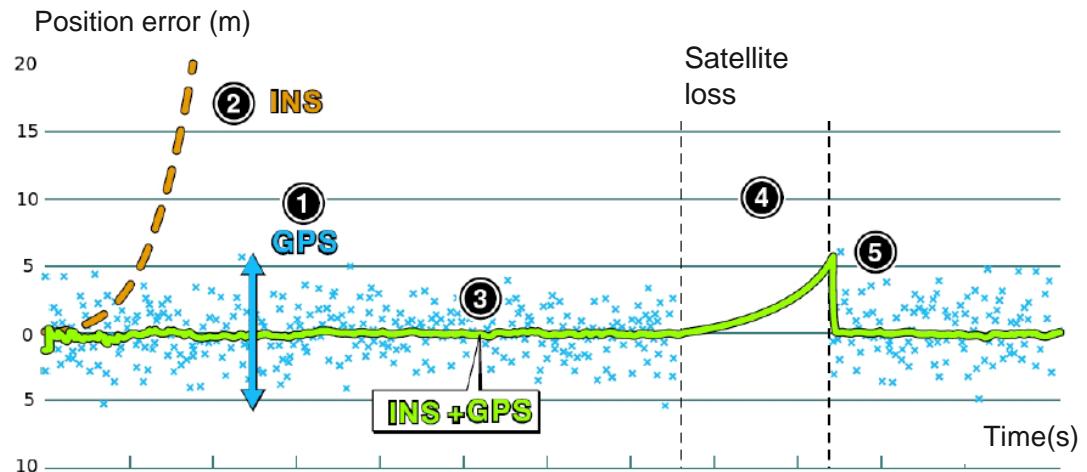


IxBlue Phins 6000 Applied Navigation Spacial Fog

INESC TEC
Embedded INS

GNSS+INS Integration

- GNSS fix correct INS drift
- INS provides navigation in case of satellite loss
- High rate from INS
- Multiple architectures of integration depending on sensor fusion level



GPS/INS integration

- GPS/ INS integration architecture depends on the type and level of data fusion algorithms
- Integration architectures [1],[2],[3]
 - Uncoupled
 - Loosely coupled
 - Tight coupled
 - Ultra-tight coupled

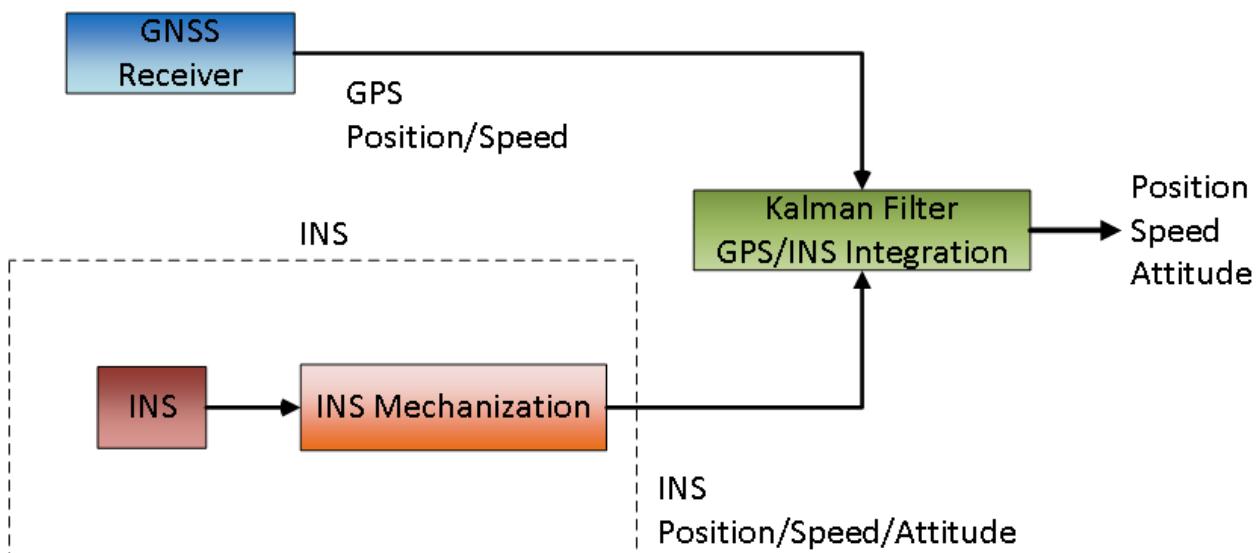
[1] David H. Titterton and John L. Weston. Strapdown Inertial Navigation Technology, The Institution of Electrical Engineers, second edition, 2004

[2] Gerard Lachapelle Andreas Wieser, Demoz Gebre-Egziabher and Mark Petovello. “Weighting GNSS Observations and Variations of GNSS/INS Integration”. InsideGNSS, pages 26–33, 2007

[3] Antonio Angrisano, “GNSS/INS Integration Methods”, 2010

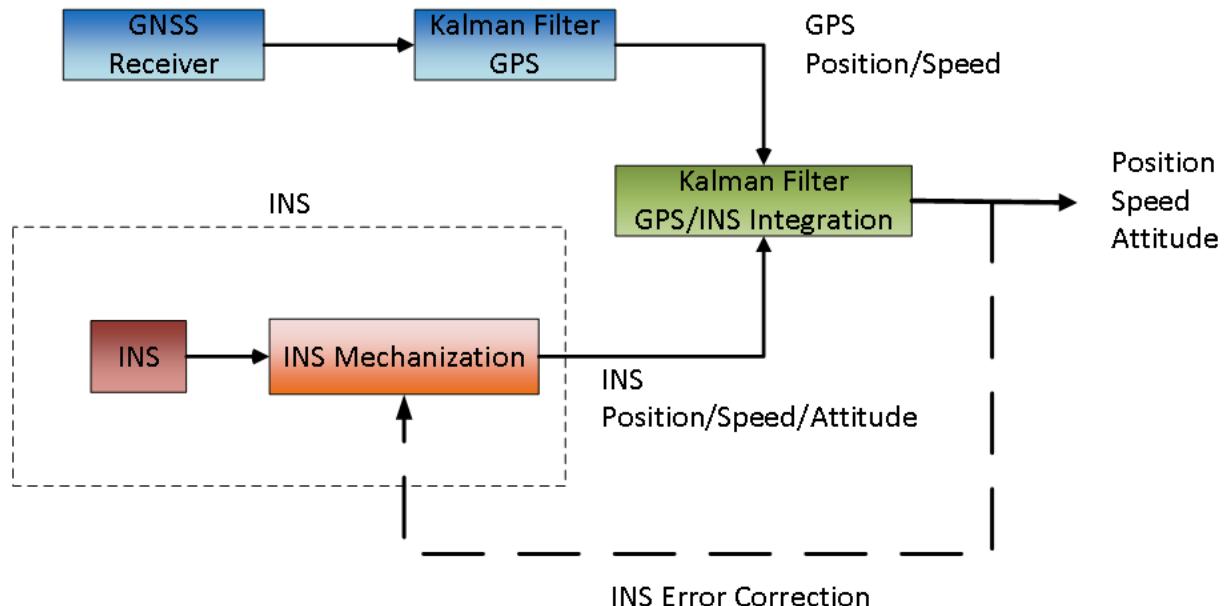
GNSS/INS Integration

- Uncoupled
 - Separate systems
 - Filter integrating outputs of each system
 - Both can operate standalone



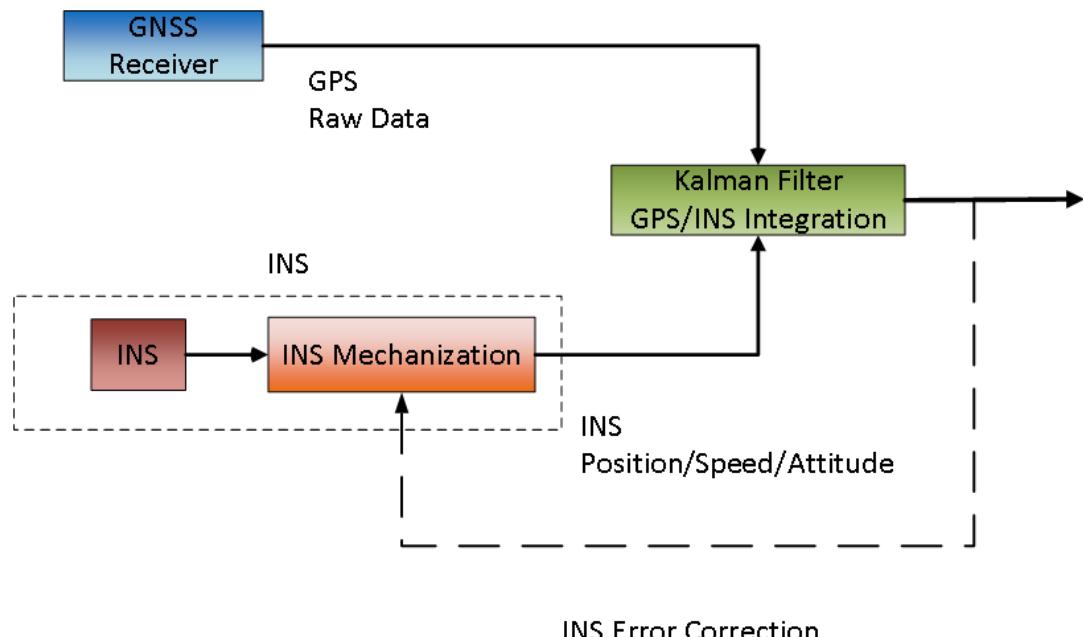
GNSS/INS Integration

- Loosely coupled
 - Standalone GNSS
 - Output of fusion filter used in te INS mechanization
 - Requires access to mechanization in INS



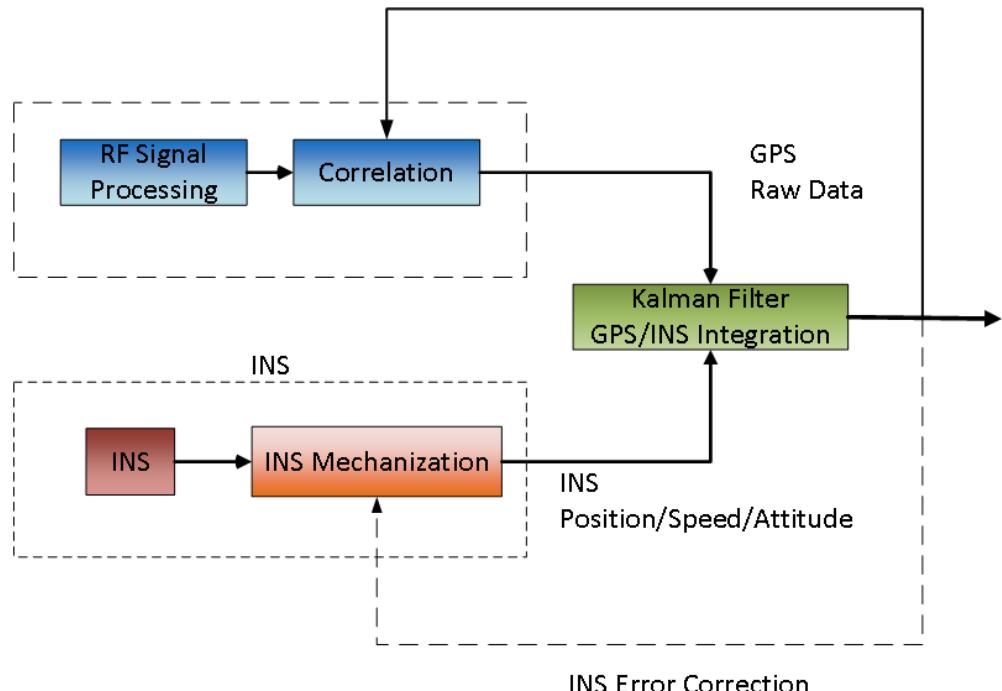
GNSS-INS integration

- Tightly coupled
 - Highly integrated fusion
 - GNSS raw data (pseudoranges, phase data) integrated with INS output
 - Can use data from less than 4 satellites
 - Estimated data used in the mechanization process
 - High quality output



GNSS/INS integration

- Ultra tight coupled
 - Most tight integration
 - GNSS raw radio signals are used (signal correlation is affected by the filter with INS information and also feed to it)
 - Requires direct access to the radio processing (usually not available)
 - Filter output feedback to INS mechanization
 - Filter complex and difficult to tune





AIS – Automatic Identification System

- External Ship tracking for legal, security, and marine traffic management
- Legal mandated identification system installed in all relevant marine vessels (all passenger ships and above 300 ton)
- AIS transmitter
 - Broadcasts ship identification, position and navigation details
 - VHF transceiver
 - Positioning system (GPS or Loran-C) and gyrocompass
 - 2-10 sec broadcasts (in transit) or 3min interval (anchored)
- Non encrypted communications, subject to spoofing
- Frequently small ships turn it off when not desiring to be located (ex: fishing vessels in non authorized areas)



Image snapshot from www.marinetraffic.com



- AIS transmitters tracked by base stations on shore or satellite
- Two classes
 - A – Large commercial vessels (integrated display, 12.5W transmission, SOTDMA, ...)
 - B – Leisure or lighter commercial (2W, CSTDMA or SOTDMA,...)
- AIS transmitter
 - 74 Km range
 - Broadcasts ship identification, position and navigation details
 - VHF transceiver
 - Positioning system (GPS or Loran-C) and gyrocompass
 - 2-10 sec broadcasts (in transit) or 3min interval (anchored)



Class A
www.raymarine.eu



Class B
www.garmin.com

Receiver,
www.em-trak.com





Underwater vehicle localization

- Problems : No GPS!, No Radio!
- Harsh and limiting environment
- Sensors for underwater localization
 - Pressure sensor
 - INS
 - Magnetometers
 - DVL sonars
 - Profilling sonars
 - Imaging sonars
 - Acoustic positioning systems
 - Cameras
 - Structured light vision systems
 - LIDAR ???



Underwater navigation instruments

INSTRUMENT	VARIABLE	INTERNAL?	UPDATE RATE	PRECISION	RANGE	DRIFT
Acoustic Altimeter	Z – Altitude	yes	varies: 0.1–10 Hz	0.01–1.0 m	varies	—
Pressure Sensor	Z – Depth	yes	medium: 1 Hz	0.01%	full-ocean	—
Inclinometer	Roll & Pitch	yes	fast: 1–10 Hz	0.1–1°	±45°	—
Magnetic Compass	Heading	yes	medium: 1–2 Hz	1–10°	360°	—
Gyro Compass	Heading	yes	fast: 1–10 Hz	0.1°	360°	10°/h
Ring-Laser Gyro	Heading	yes	fast: 1–1000 Hz	0.0018°	360°	0.44°/h
Bottom-Lock Doppler	XYZ – Velocity	yes	fast: 1–5 Hz	0.2–1.0%	30–200 m	—
12 kHz LBL	XYZ – Position	NO	varies: 0.1–1.0 Hz	0.01–10 m	5–10 km	—
300 kHz LBL	XYZ – Position	NO	1.0–5.0 Hz	±0.002 m typical	100 m	—

From: Ryan Eustice , “Large-Area Visually Augmented Navigation for Autonomous Underwater Vehicles”, PhD. Thesis, MIT, 2005



Underwater robot localization

- Infra-structured support
 - Absolute positioning, error bounding
 - Acoustic beacon support (LBL,SBL,USBL)
 - Fixed or moving beacons
 - Limited operation areas
 - High logistics and operational cost
- Non infra-structured
 - Dead reckoning sensors (IMU, DVL)
 - Terrain based / SLAM
 - Drifting position
 - Flexibility in area of operation
 - Allows stealth operation (security and military applications)
- Possible GPS fixes when at surface



AUV Navigation

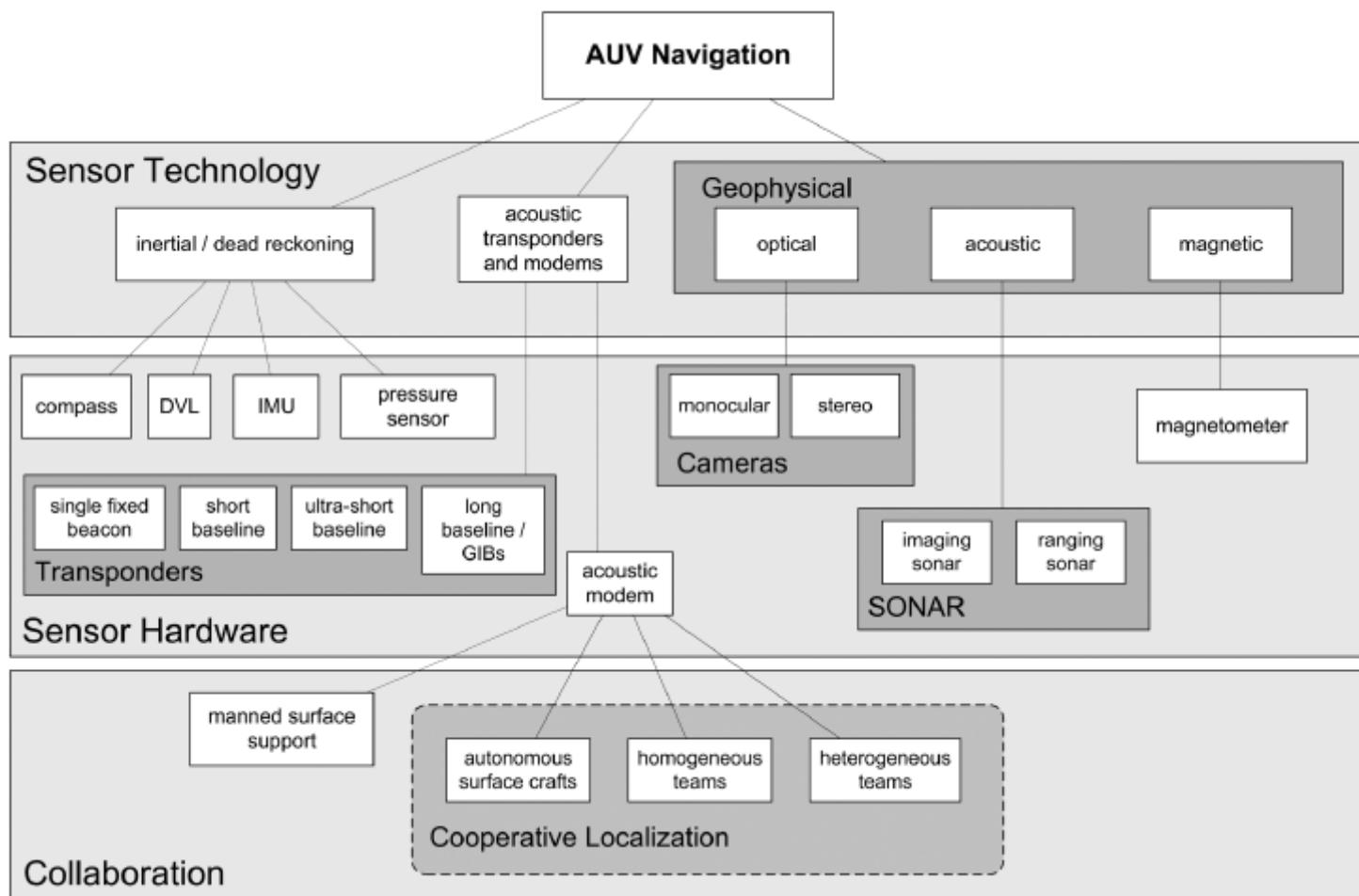


Figure from [1]

[1] L. Paull, S. Saeedi, M. Seto, and H. Li, "AUV Navigation and Localization: A Review," *IEEE J. Ocean. Eng.*, vol. 39, no. 1, pp. 131–149, Jan. 2014.

Depth

- Easy direct measurement of depth from pressure sensors (limited absolute error)
- 1 bar for each 10m of water column
- Initial calibration required
- Pressure sensor output depends on temperature
- Depends on water density (on the column) and on atmospheric pressure
- Analog or digital outputs
- Can be integrated in other sensors (ex: CTD for oceanographic applications, DVL, etc)
- High precision sensor typical accuracy and precision
 - Accuracy 0.002% FS
 - Precision 0.05% FS



Model 33x from Keller Druck



Other sensors

- Inclinometer
 - Measures roll and pitch angles
 - Slow dynamics
- Magnetic compass
 - Measures earth magnetic field – yaw angle
 - Sensitive to electromagnetic perturbations
 - For underwater vehicles operating in open sea relatively reliable



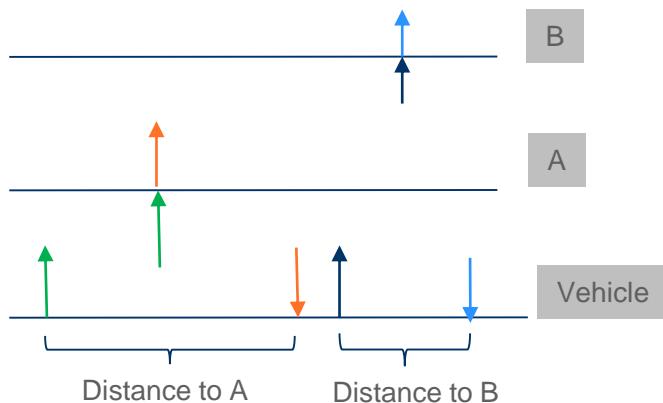
Acoustic localization systems

- Based on acoustic ping signals providing range measurements (or bearing)
- Synchronous or asynchronous
 - **TWTT / TOA – Two Way Time Travel /Time Of Arrival /Spherical Nav**
 - Distance to transponder obtained from the difference time from pinging to the receiving of acknowledgement
 - **OWTT /TDOA – One Way Time Travel /Time Difference of Arrival/ Hyperbolic Nav**
 - Distance inferred by the time of arrival of transponder signals (with synchronization, less flexible)
- External localization (vehicle tracking) / vehicle self-localization
- Update rate dependent on distance
- Dependence on acoustic conditions
 - Water sound speed (~1500 m/s) variable with temp or salinity
 - Multipath
 - Noise
- With or without communication to the vehicle (ROVs, acoustic communications)



Spherical navigation

- Location determined by trilateration
- Distances to known beacons are determined by the time of arrival of acknowledge from each beacon
- Points of constant return time lie in a sphere
- Intersection of spherical surfaces gives position of the vehicle
- Do not require synchronized clocks



Hyperbolic Navigation

- Localization process where the vehicle determines its position from the time difference of arrival of signals from known beacons
- The points in hyperbolic surface have the same time difference of arrival (relative phase)
- Requires time synchronization of emitting signals
- Phase ambiguity issues
- Loran C, and GPS examples of these type of navigation systems

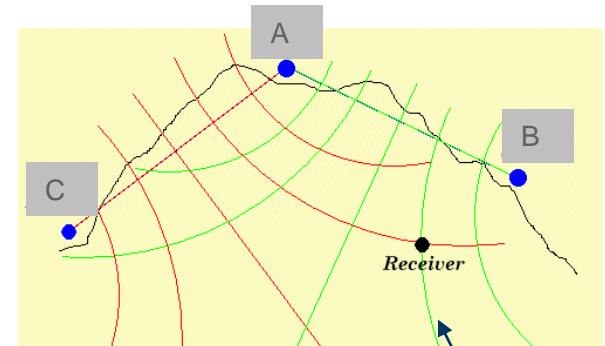
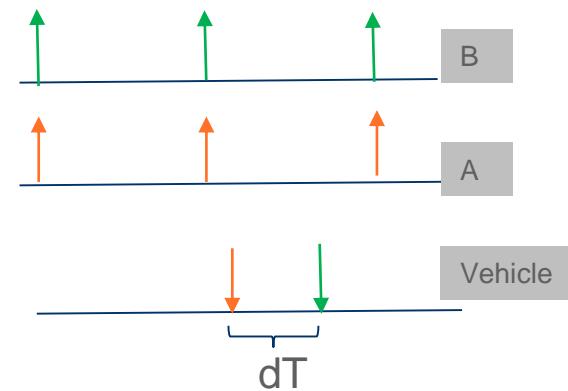


Figure adapted from www.alancordwell.co.uk

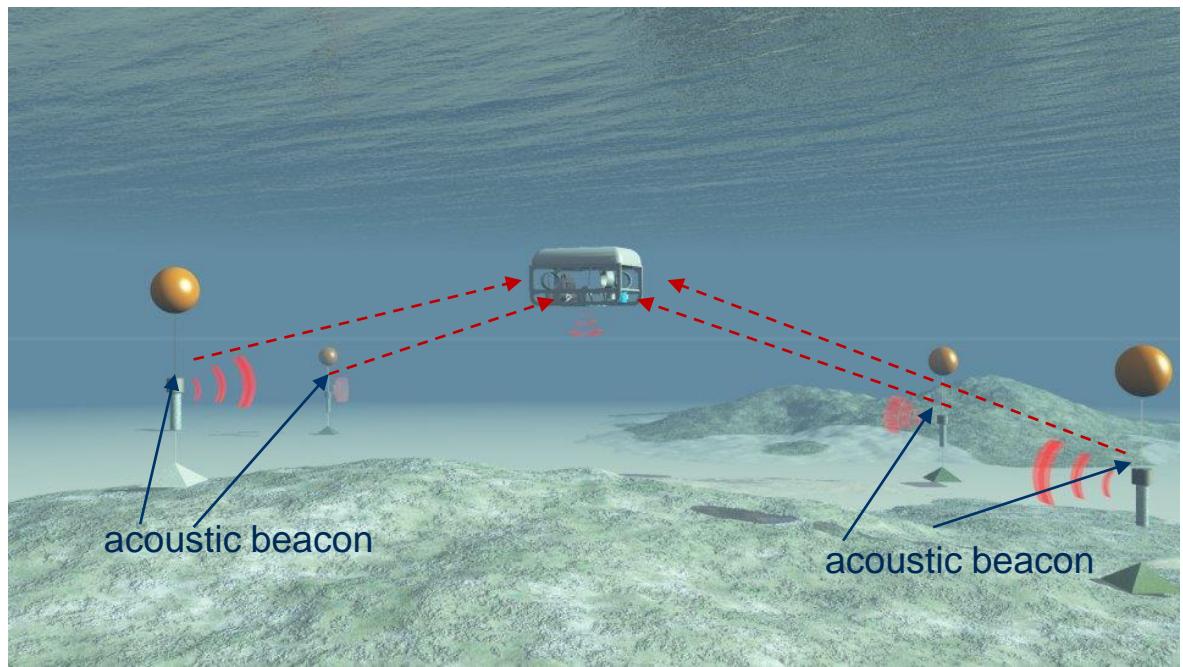
Lines of constant dT from C and Master





LBL – Long Baseline Acoustic Navigation

- Set of known acoustic beacons and acoustic transponder on vehicle
- Vehicle interrogates each beacon and range is determined by the two-way time of travel (TWTT)
- Requires environment infra-structuring and known beacon position
- Trilateration based system





Three beacon exact trilateration

3 spheres

$$r_1^2 = x^2 + y^2 + z^2$$

$$r_2^2 = (x - d)^2 + y^2 + z^2$$

$$r_3^2 = (x - i)^2 + (y - j)^2 + z^2$$

subtracting 1st eq in 2nd eq

$$x = \frac{r_1^2 - r_2^2 + d^2}{2d}$$

substituting in 1st eq, intersection is obtained (circumference)

$$y^2 + z^2 = r_1^2 - \frac{(r_1^2 - r_2^2 + d^2)^2}{4d^2}$$

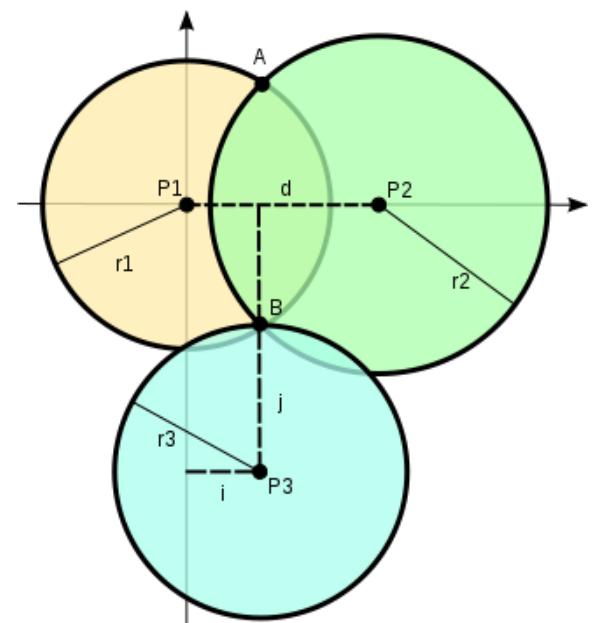
substituting $y^2 + z^2 = r_1^2 - x^2$ in the 3rd sphere and resolving in order to y

$$y = \frac{r_1^2 - r_3^2 - x^2 + (x - i)^2 + j^2}{2j} = \frac{r_1^2 - r_3^2 + i^2 + j^2}{2j} - \frac{i}{j}x$$

now with x, and y known, manipulating 1st eq

$$z = \pm \sqrt{r_1^2 - x^2 - y^2}$$

Trilateration



What if there are more beacons?

Trilateration – Unconstrained Least Squares

- A simple trilateration algorithm with measurement noise [1]
- Vehicle assumed to be static

$\mathbf{p} \in \mathbb{R}^n$ Vehicle position

$\mathbf{p}_i \in \mathbb{R}^n \quad i \in \{1, 2, \dots, m\}$

$\mathbf{P} = [\mathbf{p}_1 \dots \mathbf{p}_m] \in \mathbb{R}^{n \times m}$ Landmark positions

$$\bar{r}_i = \|\mathbf{p} - \mathbf{p}_i\| + w_i, \quad i \in \{1, 2, \dots, m\}$$

Range measurement
to landmark i

$$\underbrace{r_i}_{\text{Range measurement to landmark i}}$$

Measurement
noise

$$\mathbf{r} = [r_1, \dots, r_m]^T \quad \mathbf{w} = [w_1, \dots, w_m]^T$$

$$\bar{\mathbf{r}} = [\bar{r}_1, \dots, \bar{r}_m]^T$$

$$\mathbf{R} = \mathbb{E} \{ \mathbf{w} \mathbf{w}^T \} \in \mathbb{R}^{m \times m}$$
 Measurement error
covariance matrix

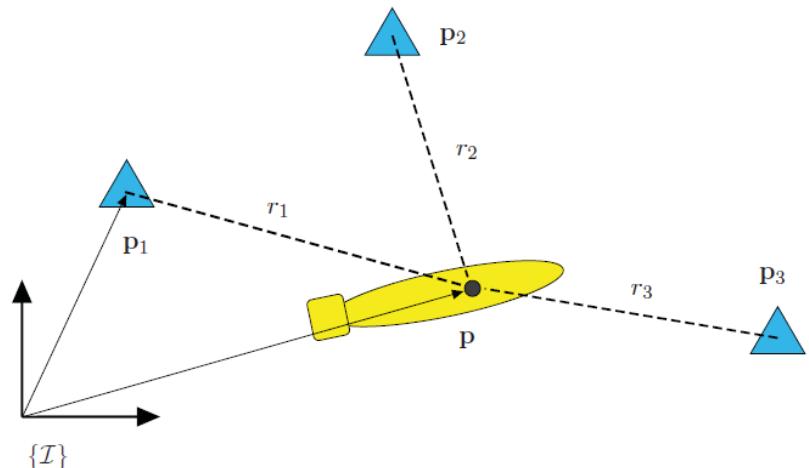


Figure from [1]

PROBLEM

Compute estimate $\hat{\mathbf{p}} \in \mathbb{R}^n$ of \mathbf{p}
given measurements $\bar{\mathbf{r}} = \mathbf{r} + \mathbf{w}$ with \mathbf{W} a zero mean

Gaussian error with covariance $\mathbf{R} \in \mathbb{R}^{m \times m}$

[1] A. Alcocer, "Positioning and Navigation Systems for Robotic Underwater Systems", PhD. Thesis, IST, 2009.

Trilateration – Unconstrained Least Squares

- Manipulating the squared measurements in order to obtain a linear equation in the position and its square norm, a linear LS can be solved (ignoring dependence of position on the norm) [1],[2].

$$d_i = r_i^2 = \|\mathbf{p} - \mathbf{p}_i\|^2 = (\mathbf{p} - \mathbf{p}_i)^T(\mathbf{p} - \mathbf{p}_i) = \mathbf{p}^T\mathbf{p} - 2\mathbf{p}_i^T\mathbf{p} + \mathbf{p}_i^T\mathbf{p}_i \quad \text{Square of distance to landmark } i$$

Square of distance measurements

$$\bar{d}_i := \bar{r}_i^2 = (r_i + w_i)^2 = r_i^2 + 2r_i w_i + w_i^2 = d_i + 2r_i w_i + w_i^2 = d_i + \xi_i \quad \text{Error in square distance measurement}$$

$$\bar{\mathbf{d}} = \mathbf{d} + \boldsymbol{\xi}$$

Square of distances

$$\begin{aligned} \mathbf{d} &= \begin{bmatrix} d_1 \\ \vdots \\ d_m \end{bmatrix} = \begin{bmatrix} \mathbf{p}^T\mathbf{p} - 2\mathbf{p}_1^T\mathbf{p} + \mathbf{p}_1^T\mathbf{p}_1 \\ \vdots \\ \mathbf{p}^T\mathbf{p} - 2\mathbf{p}_m^T\mathbf{p} + \mathbf{p}_m^T\mathbf{p}_m \end{bmatrix} = \|\mathbf{p}\|^2 \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} - 2 \begin{bmatrix} \mathbf{p}_1^T \\ \vdots \\ \mathbf{p}_m^T \end{bmatrix} \mathbf{p} + \begin{bmatrix} \mathbf{p}_1^T\mathbf{p}_1 \\ \vdots \\ \mathbf{p}_m^T\mathbf{p}_m \end{bmatrix} \\ &= \|\mathbf{p}\|^2 \mathbf{1}_m - 2\mathbf{P}^T\mathbf{p} + \delta(\mathbf{P}^T\mathbf{P}) \end{aligned}$$

[1] A. Alcocer, “Positioning and Navigation Systems for Robotic Underwater Systems”, PhD. Thesis, IST, 2009.

[2] K. Cheung et al, “Least squares algorithms for time-of-arrival based mobile location”, IEEE Trans. On Acoustics, Speech and Singnal processing, vol2, pp145-148,2004



Trilateration – Unconstrained Least Squares

With the $\delta(\cdot)$ operator defined as: $\delta(\mathbf{A}) = \delta\left(\begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & \ddots & \vdots \\ a_{n1} & \dots & a_{nn} \end{bmatrix}\right) = \begin{bmatrix} a_{11} \\ \vdots \\ a_{nn} \end{bmatrix} \in \mathbb{R}^n$ $\delta(\mathbf{b}) = \delta\left(\begin{bmatrix} b_1 \\ \vdots \\ b_n \end{bmatrix}\right) = \begin{bmatrix} b_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & b_n \end{bmatrix} \in \mathbb{R}^{n \times n}$

Reorganizing $\mathbf{d} = \|\mathbf{p}\|^2 \mathbf{1}_m - 2\mathbf{P}^T \mathbf{p} + \delta(\mathbf{P}^T \mathbf{P})$ It comes: $2\mathbf{P}^T \mathbf{p} - \|\mathbf{p}\|^2 \mathbf{1}_m = \delta(\mathbf{P}^T \mathbf{P}) - \bar{\mathbf{d}} + \xi$ (*)

$$\bar{\mathbf{d}} = \mathbf{d} + \xi$$

(*) Can be written in linear form

$$\boxed{\mathbf{A}\boldsymbol{\theta} = \mathbf{b} + \xi}$$

with $\mathbf{A} = \begin{bmatrix} 2\mathbf{P}^T & -\mathbf{1}_m \end{bmatrix} = \begin{bmatrix} 2\mathbf{p}_1^T & -1 \\ \vdots & \vdots \\ 2\mathbf{p}_m^T & -1 \end{bmatrix}$

$$\begin{bmatrix} 2\mathbf{P}^T & -\mathbf{1}_m \end{bmatrix} \begin{bmatrix} \mathbf{p} \\ \|\mathbf{p}\|^2 \end{bmatrix} = \delta(\mathbf{P}^T \mathbf{P}) - \bar{\mathbf{d}} + \xi$$

\mathbf{A} $\boldsymbol{\theta}$ \mathbf{b} ξ

Note that all unknowns are in $\boldsymbol{\theta}$

solving for the unknown neglecting the constraints between \mathbf{p} and $\|\mathbf{p}\|^2$

$$\mathbf{b} = \delta(\mathbf{P}^T \mathbf{P}) - \bar{\mathbf{d}} = \begin{bmatrix} \|\mathbf{p}_1\|^2 - \bar{d}_1 \\ \vdots \\ \|\mathbf{p}_m\|^2 - \bar{d}_m \end{bmatrix}$$

$$\boldsymbol{\theta} = \begin{bmatrix} \mathbf{p} \\ \|\mathbf{p}\|^2 \end{bmatrix}$$

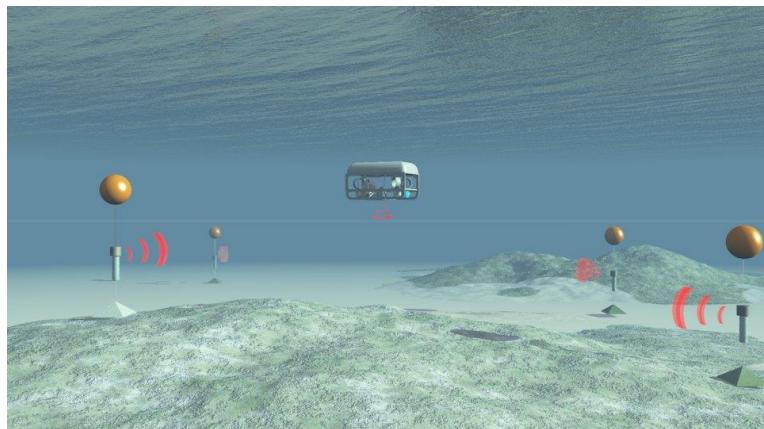
$$\boldsymbol{\theta}^* = \arg \min_{\boldsymbol{\theta} \in \mathbb{R}^{n+1}} \|\mathbf{A}\boldsymbol{\theta} - \mathbf{b}\|^2 \rightarrow$$

$$\boxed{\boldsymbol{\theta}^* = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{b}}$$

[1] A. Alcocer, "Positioning and Navigation Systems for Robotic Underwater Systems", PhD. Thesis, IST, 2009.



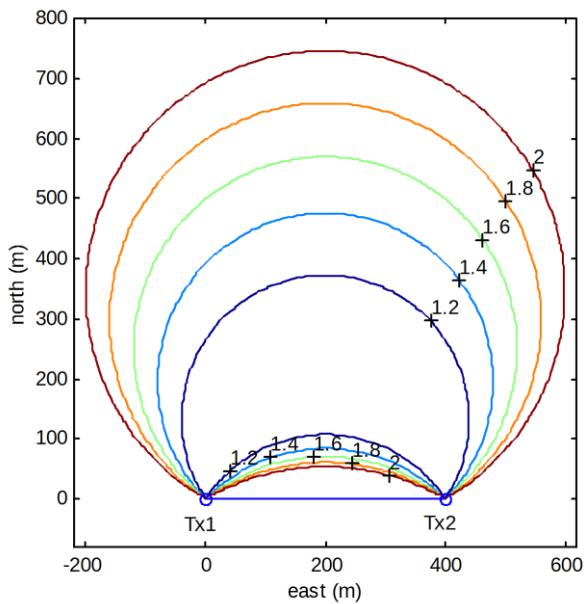
- Beacons
 - On sea bottom
 - Moored with marking buoys on the surface
 - On moving surface buoys/ASVs (moving baseline)
- Many commercial solutions incorporate acoustic communications
- High operational costs
 - Deployment from the surface
 - Calibration
 - Recovery



EvoLogics and Kongsberg LBL nodes



- Limited area of operation (few km for low freq.)
- Relatively low ping rates
 - 10 sec ping cycles
- Errors (low freq)
 - HIPAP USBL, LBL 7-15kHz
 - Range accuracy 0.2% (no ray bending ...)
 - 6m @ 3000m
 - Ray bending can increase error to 10s of meters...
- Errors (high freq)
 - LBL 300KHz (EXACT)
 - Few centimeter
 - Limited range (100's m)
- Error sensitive to range and position
- Beacon location requires calibration
 - Beacon interrogation from multiple locations at surface to trilaterate its exact location
 - Calibration errors impact directly on navigation solution





Intelligent Buoys*

- Surface buoys
 - Acoustic transceiver
 - GPS receiver (position and time synchronization)
 - Radio communications (allowing real time vehicle tracking)
 - Known position
- Can support multiple acoustic localization methods (LBL or inverted USBL)
- No deep water installation required
- Position available at surface

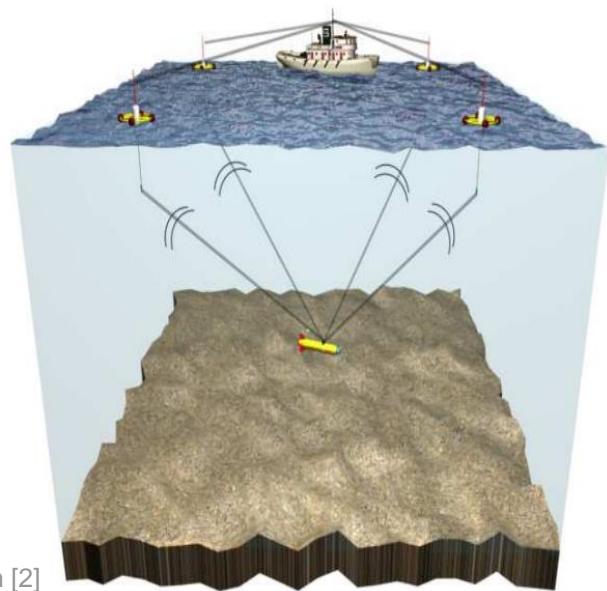
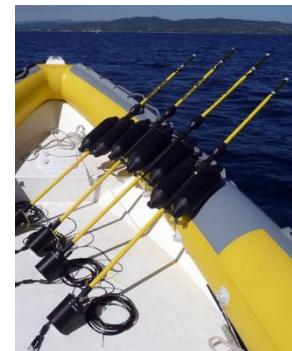


Image from [2]



ACSA (Spain) GIBs



INESC TEC Acoustic location buoy

*Also known as GIB Buoys – GPS Intelligent Buoys [1]

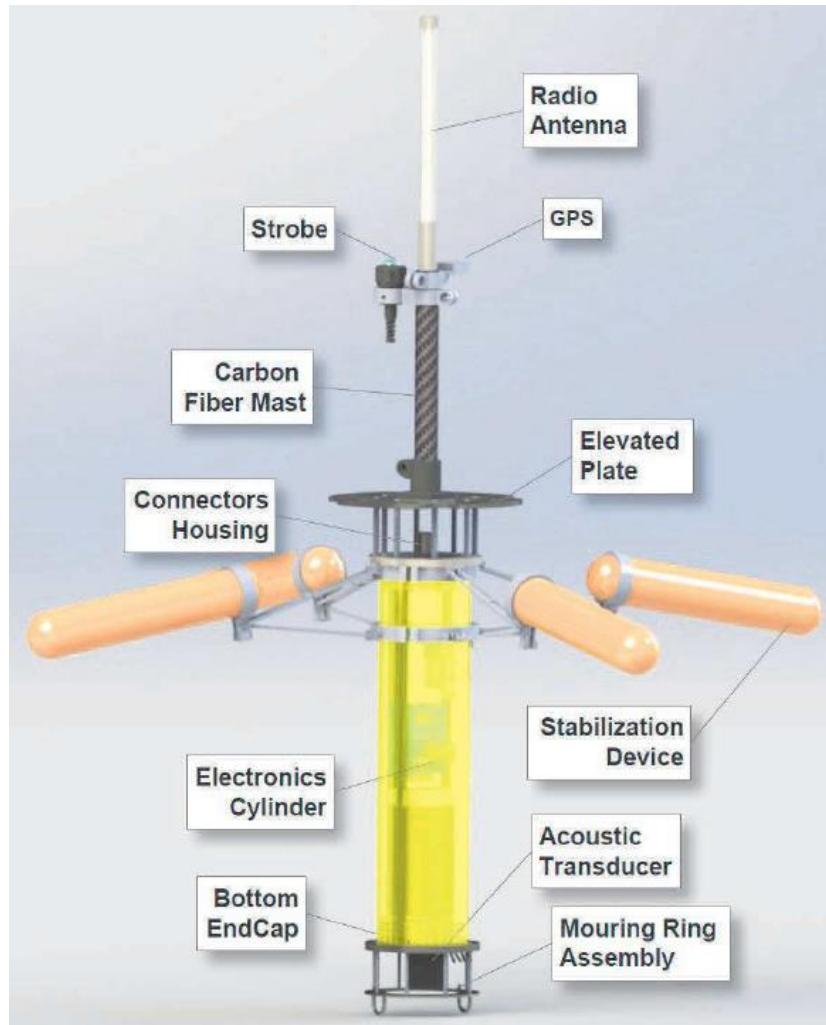
[1] Thomas, H.G. "GIB Buoys: An Interface Between Space and Depths of the Ocean", Proceedings of IEEE Autonomous Underwater Vehicles, Cambridge, MA, USA, pages 181-184, August 1998

[2] A. Alcocer, "Positioning and Navigation Systems for Robotic Underwater Systems", PhD. Thesis, IST, 2009

Intelligent Buoys



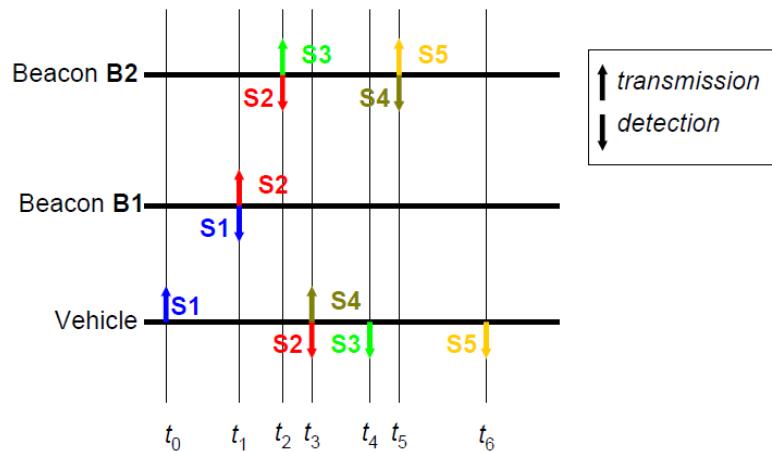
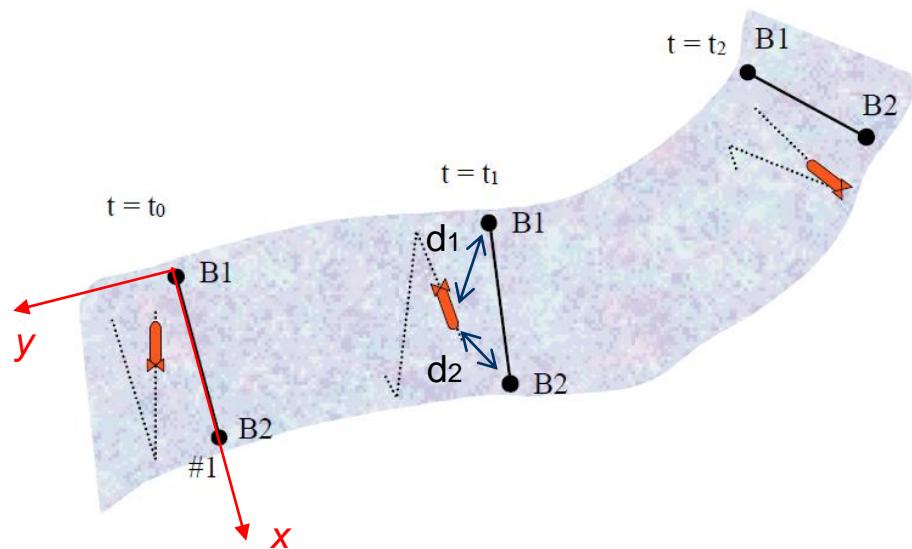
INESC TEC Acoustic
Nav. Buoy



[1] R. Almeida et al., "Man portable acoustic navigation buoys", Proceedings of IEEE Oceans Shanghai 2016

Moving Baseline

- Example [1], AUV navigating in a moving baseline
- TWTT asynchronous



B2 responds also to B1 response S2 with S3, vehicle receives this at t4, allowing for determination:

$$m = d_1 + d_2 + l$$

With the values obtained by the time differences

$$x = \frac{d_1^2 - d_2^2 + l^2}{2l}$$

$$y = \sqrt{d_1^2 - x^2}$$

$$d_1 = \frac{t_3 - t_0}{2c}$$

$$m = \frac{t_4 - t_0}{c}$$

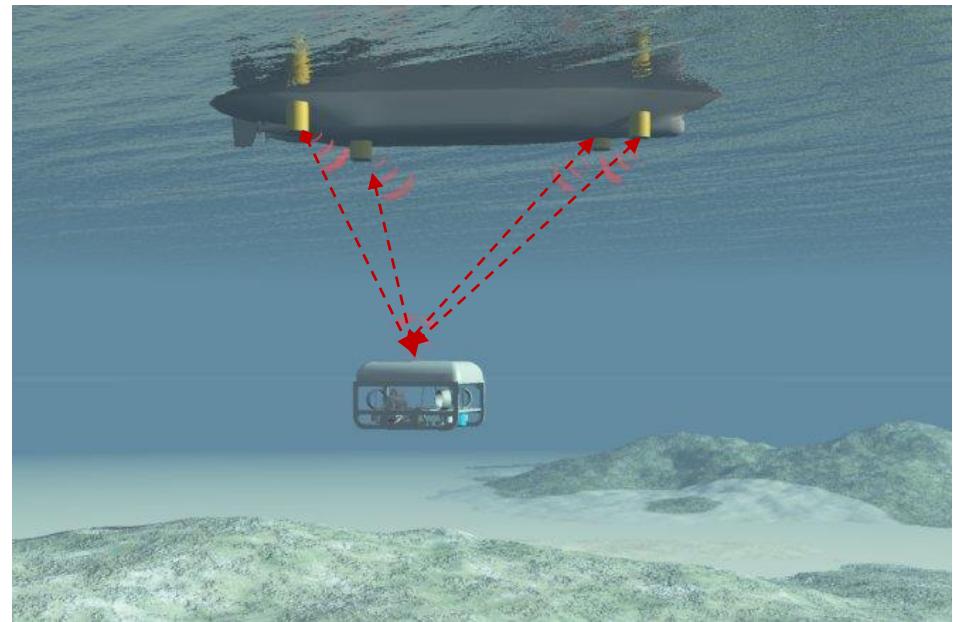
$$d_2 = \frac{t_6 - t_3}{2c}$$

[1] A. Matos, N. Cruz "AUV navigation and guidance in a moving acoustic network," MTS/IEEE Oceans 2005.



SBL- Short Baseline Navigation

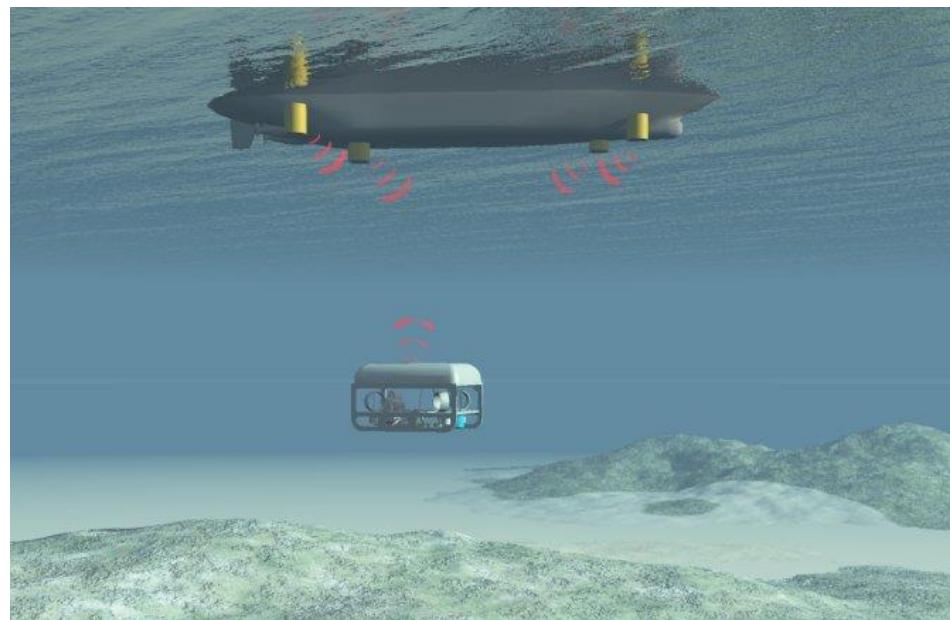
- Shorter baseline than LBL (a few meters)
- Beacons on surface support ship hull or infrastructure
- Beacon position already pre-calibrated
- Shorter baseline implying larger errors
- Does not allow ships of opportunity
- High costs for ship installation





SBL-Short Baseline Navigation

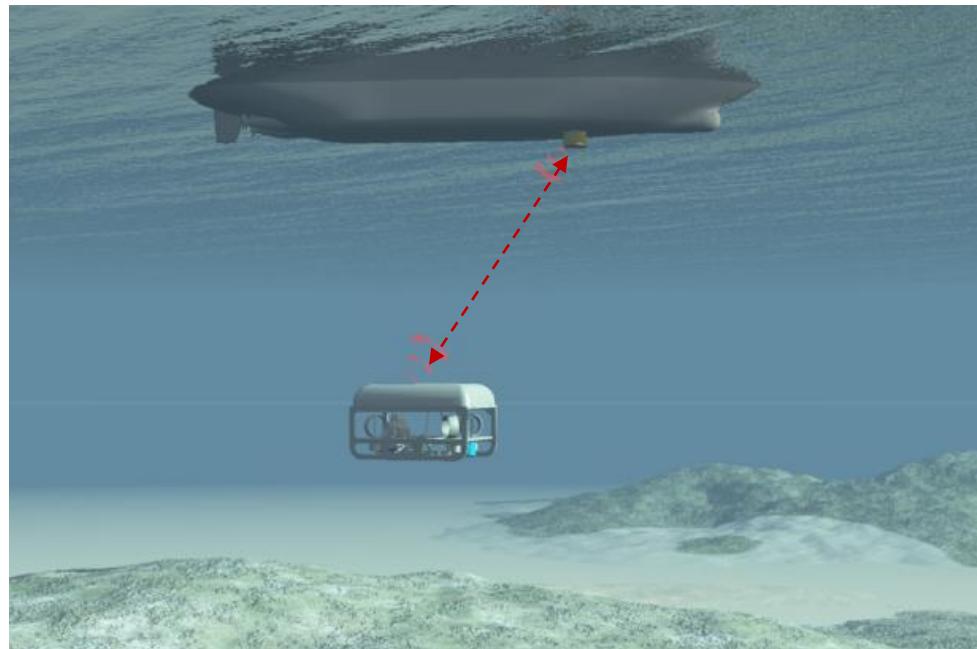
- Two possible architectures
 - Beacon based – TDOA – Uses time differences at the receiver (surface ship) to determine range and bearing
 - Transponder based – TOA – Uses





USBL – Ultra Short Baseline Navigation*

- Vehicle carries a transponder
- Transceiver on the surface (attached to vessel)
- Very short baseline (cm)
- Single transceiver at surface (containing multiple elements)
- USBL transceiver provides range and bearing to the target transponder
- Orientation of the transceiver required
 - Usual integration with AHRS or INS



*Also known as SSBL – Super Short Baseline



USBL-Ultra Short Baseline

- Lower cost and simpler to operate than LBL or SBL
- Can be used in vessel of opportunity
- Orientation of returning signal measured by signal phase difference
- Slant range measured by TOA (beacon mode also possible)
- Measurements available at the transceiver side
- Position estimation very sensitive to transceiver head attitude error and increased errors with slant
- For onboard AUV navigation, acoustic communications integration is also required
- Possible inverted configuration (iUSBL)
- Accuracy, Ex: HIPAP USBL, LBL 7-15kHz
 - Range accuracy 0.2% (no ray bending ...)
 - 6m @ 3000m
 - Ray bending can increase error to 10s of meters...



HiPAP 502 USBL transceiver
from Kongsberg



transponder



transceiver

USBL-Ultra Short Baseline

- Range detected by TDOA
- Orientation by phase delay at receiver array

$$\Delta t = \frac{b \cos \theta}{c} \quad \Rightarrow \quad \Delta \varphi = \underbrace{2\pi f}_{\omega} \left(\frac{b}{c} \right) \cos \theta$$

$$\omega = \frac{\Delta \varphi}{\Delta t} \Rightarrow \Delta t = \frac{\Delta \varphi}{\omega}$$

$$\theta = \cos^{-1} \left(\frac{\Delta \varphi \cdot c}{2\pi f \cdot b} \right)$$

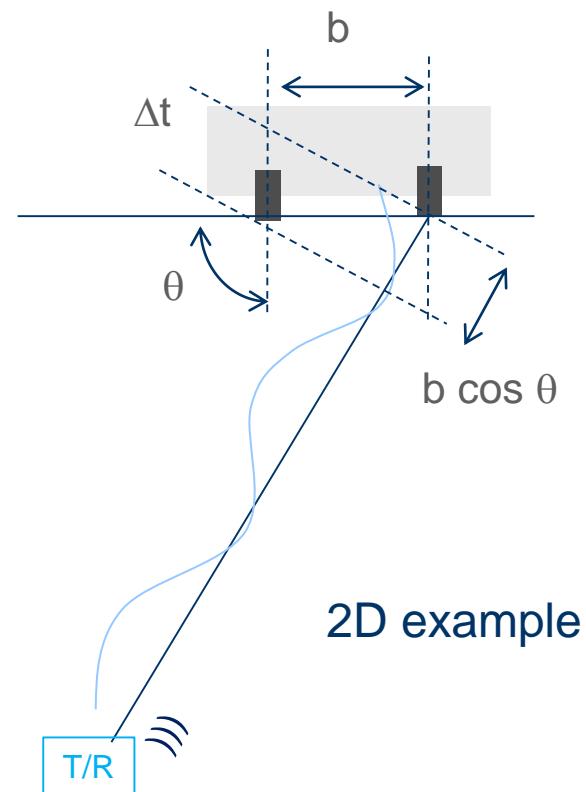


Figure adapted from [1]



DVL Navigation

- DVL provides relative velocity to the bottom (or other structures)
- Direct measuring of u , v and w
- Lower position drift compared with INS (only one integration step to obtain position)
- Limited sensor range (bottom track $<100m$, typically)
- Accuracy
 - 0.1-0.5% travelled distance



Doppler effect

- Relative velocity of transmitter and receiver alter receiving frequency

vT – distance covered by emitter in one period

cT – distance between emitter at initial position and receiver (covered by 1st wave)

$c(t-T)$ – distance from emitter to the second wave after traveling one period

λ_r - wavelength at receiver

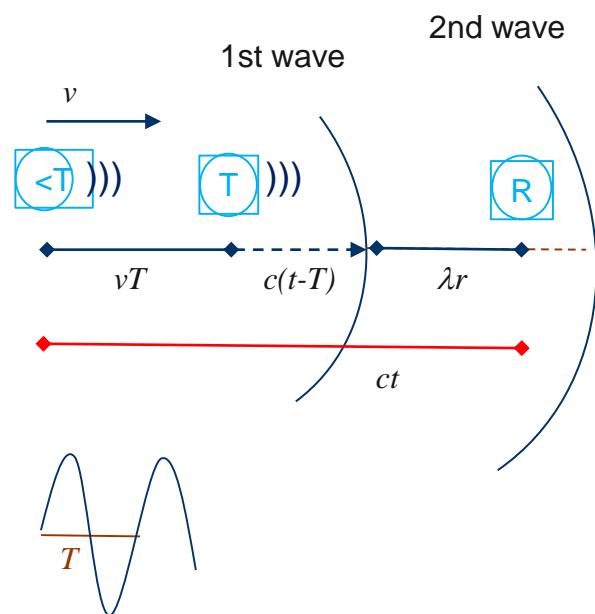
f_r - received wavelength

f_t - transmitting frequency

T - transmitting period

v - emitter velocity

c - sound velocity



$$ct = vT + c(t - T) + \lambda_r \quad \lambda_r = (c - v)T$$

$$0 = vT - cT + \lambda_r$$

$$\lambda_r = (c - v)T$$

$$T = \frac{1}{f_t} \implies \lambda_r = \frac{c-v}{f_t}$$

$$f_r \lambda_r = c \implies$$

$$f_r = f_t \left(\frac{c}{c-v} \right)$$

DVL Navigation

Transmitter moving, frequency at the stationary receiver

$$f_r = \frac{f_t}{1 \pm \frac{v}{c}}$$



Transmitter stationary, frequency at the receiver (moving)

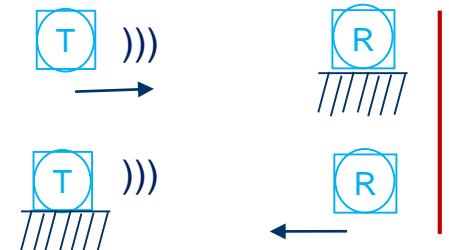
$$f_r = f_t \left(1 \mp \frac{v}{c} \right)$$



In case of DVL on a vehicle, its is a moving transmitter and receiver, intermediate reflection treated as a stationary receiver followed by a stationary transmitter

$$f_r = f_t \left(\frac{1 \mp \frac{v}{c}}{1 \pm \frac{v}{c}} \right)$$

Simplifying (v much less than c) multiplying by denoninate conjugate



Sea bottom reflection

$$\Delta f = f_r - f_t \approx \pm \frac{2v f_t}{c} \quad \Rightarrow \quad v \approx \frac{\Delta f c}{2 f_t}$$

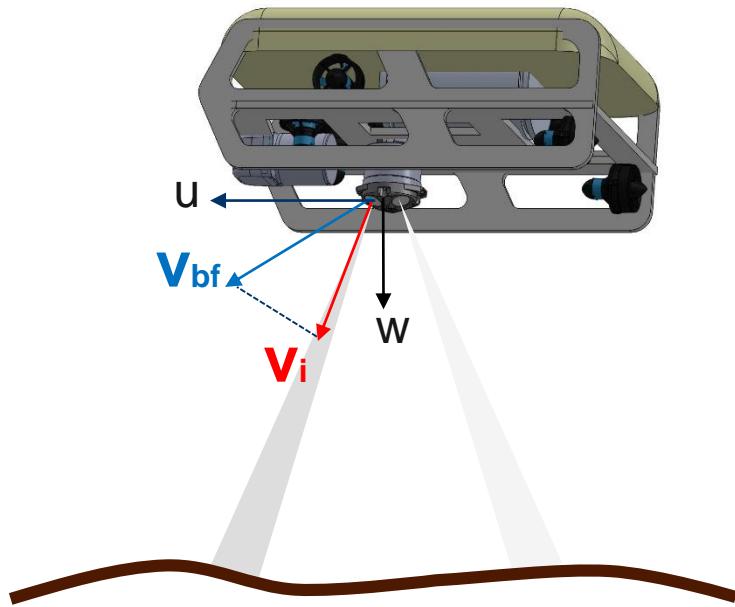
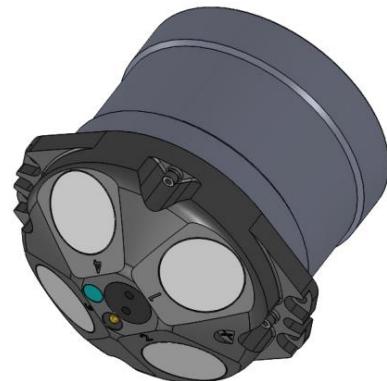
c sound velocity

[1] N. Brokloff, "Matrix algorithm for Doppler sonar navigation", MTS/IEEE Oceans 1994.

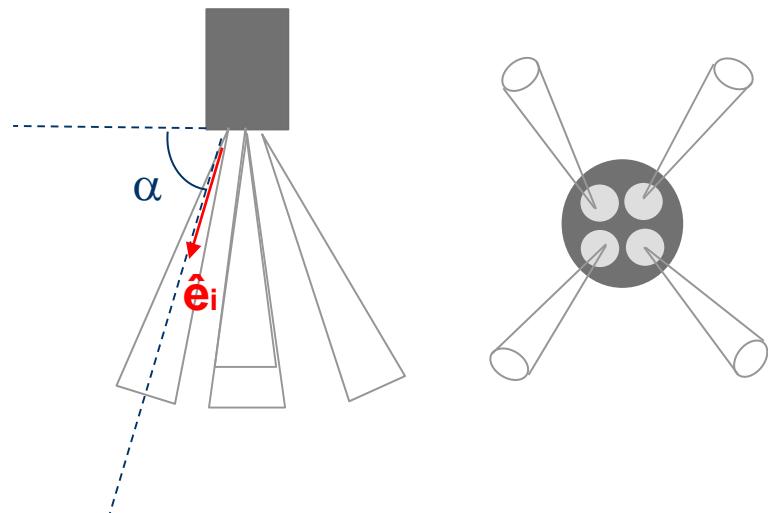


Typical 4 transducer Janus configuration

- Multiple beams provide all components of velocity (in instrument fixed frame)
- With movement in any direction at least 3 transducers move not perpendicular to acoustic signal direction of travel
- Each beam measures the velocity component on its direction (given by unit vector \hat{e}_i)



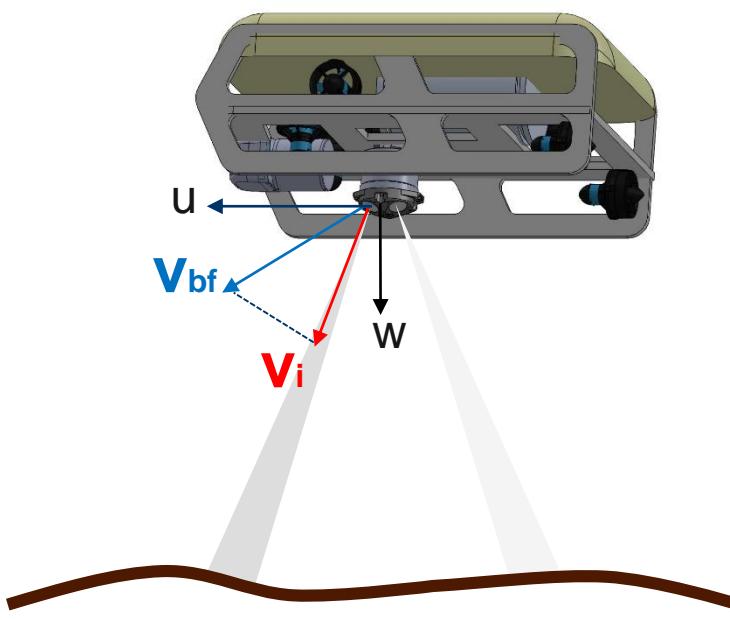
Janus configuration





DVL velocity

- Considering the instrument reference frame perfectly aligned with the body fixed frame {b}
 - Vehicle velocity v_{bf} is projected in each beam axis
 - With all 4 beams with bottom lock, solution is over determined, it can be obtained by least squares or by choosing pairs of beams



Vehicle/instrument velocity

$$\mathbf{v}_{bf} = [u \quad v \quad w]$$

Unit vectors for the four beams

$$\hat{e}_1 = [-\cos \alpha \quad 0 \quad \sin \alpha]$$

$$\hat{e}_2 = [\cos \alpha \quad 0 \quad \sin \alpha]$$

$$\hat{e}_3 = [0 \quad -\cos \alpha \quad \sin \alpha]$$

$$\hat{e}_4 = [0 \quad \cos \alpha \quad \sin \alpha]$$

Doppler shift on beam i

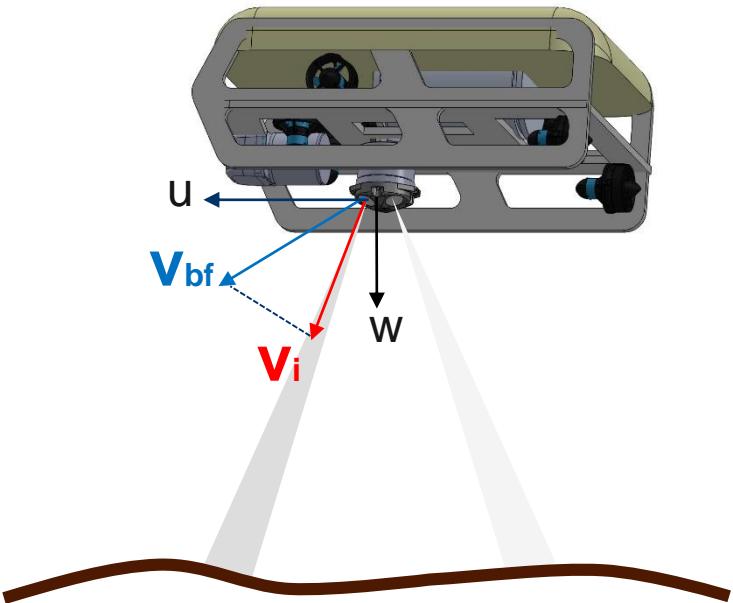
$$\Delta f_i = \frac{2f_t}{c} \hat{e}_i \cdot \mathbf{v}_{bf}$$

Velocity along beam i

$$v_i = \hat{e}_i \cdot \mathbf{v}_{bf}$$

DVL velocity

Doppler shifts in matrix form as function of velocity



$$\Delta \mathbf{f} = \begin{bmatrix} \Delta f_1 \\ \Delta f_2 \\ \Delta f_3 \\ \Delta f_4 \end{bmatrix} = \frac{2f_t}{c} \begin{bmatrix} \hat{e}_1 \\ \hat{e}_2 \\ \hat{e}_3 \\ \hat{e}_4 \end{bmatrix} \cdot \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{2f_t}{c} \mathbf{E} \mathbf{v}_{bf}$$

Pre-multiplying by \mathbf{E}^T (since \mathbf{E} is not square) and solving the least squares problem in order to \mathbf{v}_{bf}

$$\mathbf{E}^T \Delta \mathbf{f} = \frac{2f_t}{c} \mathbf{E}^T \mathbf{E} \mathbf{v}_{bf}$$

$$[\mathbf{E}^T \mathbf{E}]^{-1} \Delta \mathbf{f} = \frac{2f_t}{c} \mathbf{v}_{bf}$$

$$\mathbf{v}_{bf} = \frac{c}{2f_t} [\mathbf{E}^T \mathbf{E}]^{-1} \mathbf{E} \Delta \mathbf{f}$$



DVL navigation

Doppler sensor returns velocity in its reference frame $\mathbf{v}_{doppler}$

if sensor not aligned with $\{\mathbf{b}\}$ then $\mathbf{v}_{doppler} \neq \mathbf{v}_{bf}$

but the sensor frame **{doppler}** rotation wrt. $\{\mathbf{b}\}$ can be determined, thus

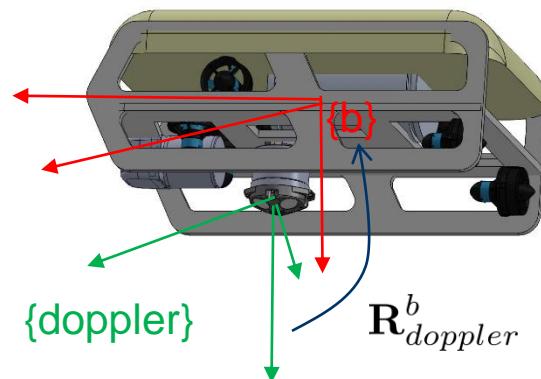
$$\mathbf{v}_{bf} = \mathbf{R}_{doppler}^b \cdot \mathbf{v}_{doppler}$$

velocities in the world frame (or NED $\{\mathbf{n}\}$) are obtained by the linear transformation

$$\mathbf{v}_n = \mathbf{R}_b^n(\phi, \theta, \psi) \cdot \mathbf{v}_{bf}$$

world or NED velocities \mathbf{v}_n are integrated to determine position

$$\mathbf{x}_n = \mathbf{x}(t_0) + \int_{t_0}^t \mathbf{v}_n(\tau) d\tau$$



Typically DVL sensors allow for coordinate conversion with external input from navigation sensors such as INS or GPS
Additional LS velocity error measure output is also commonly available



Terrain based navigation

Use information from the environment in order to locate itself

Absolute location ($\{n\}$, $\{ecef\}$ etc)

Map is needed

Relative navigation

Determination of relative position and pose (velocities) to relevant local elements in the environment (ex: relative to $\{l\}$)

Many applications do not require absolute positioning* (ex: motion relative to a wall, inspection of a ship hull)

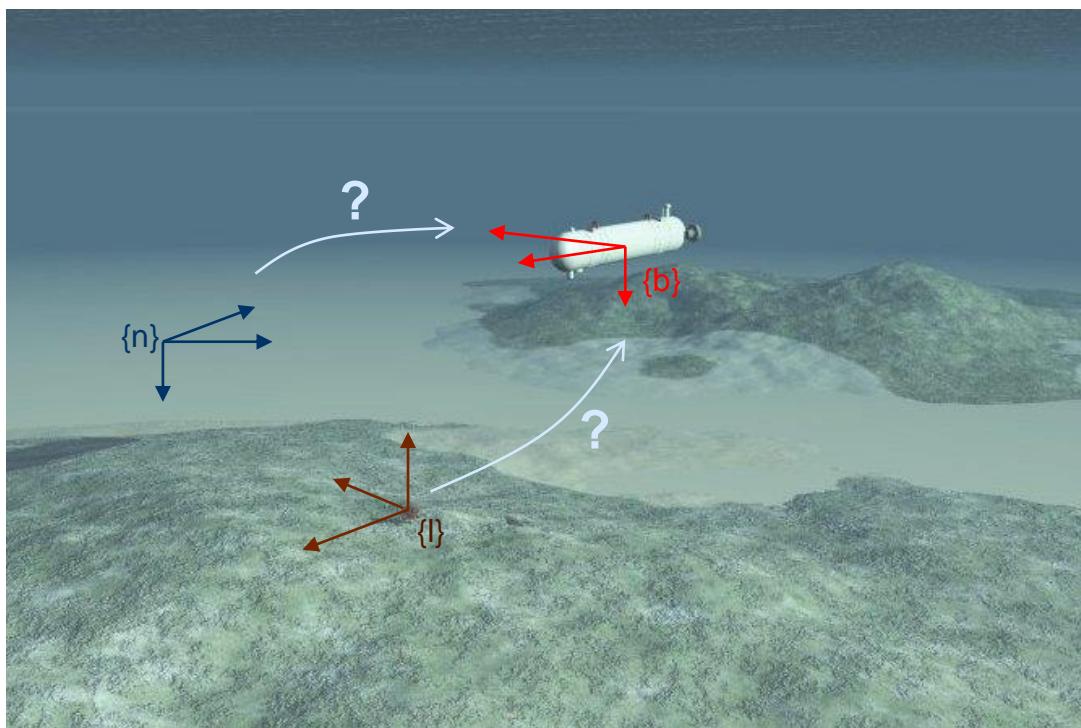
Two possibilities:

MAP

- Map based navigation

No MAP

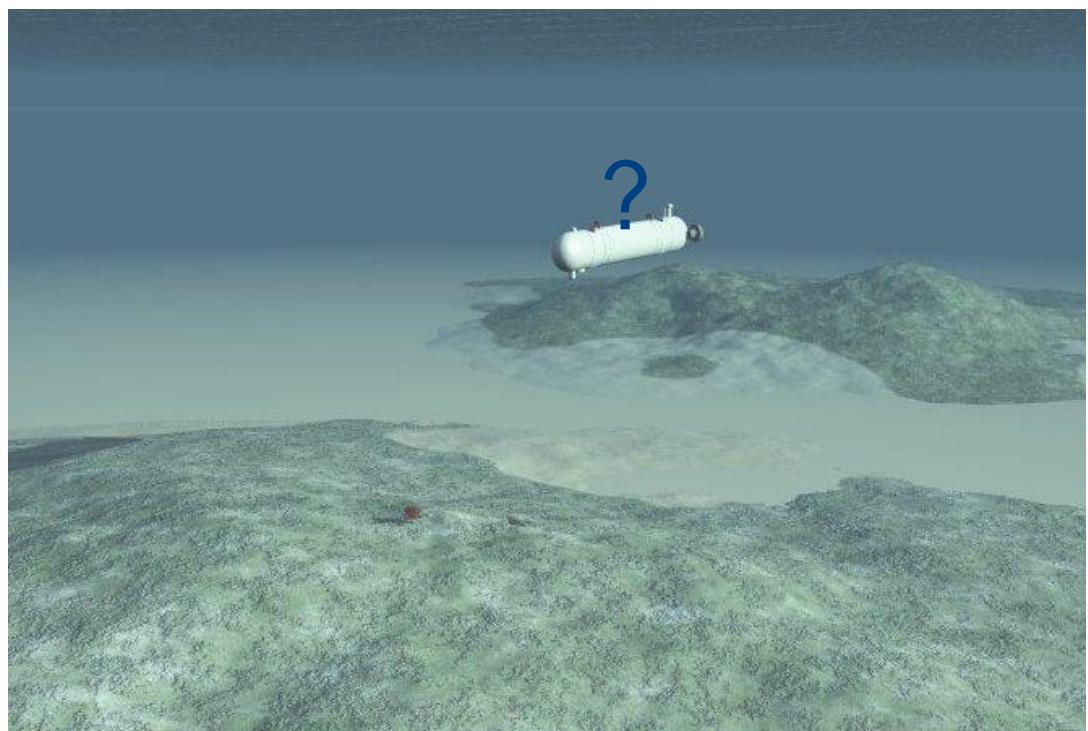
- Local relative positioning
(ex. Visual servoing)
- SLAM





Terrain based navigation

Problem: where am I ?





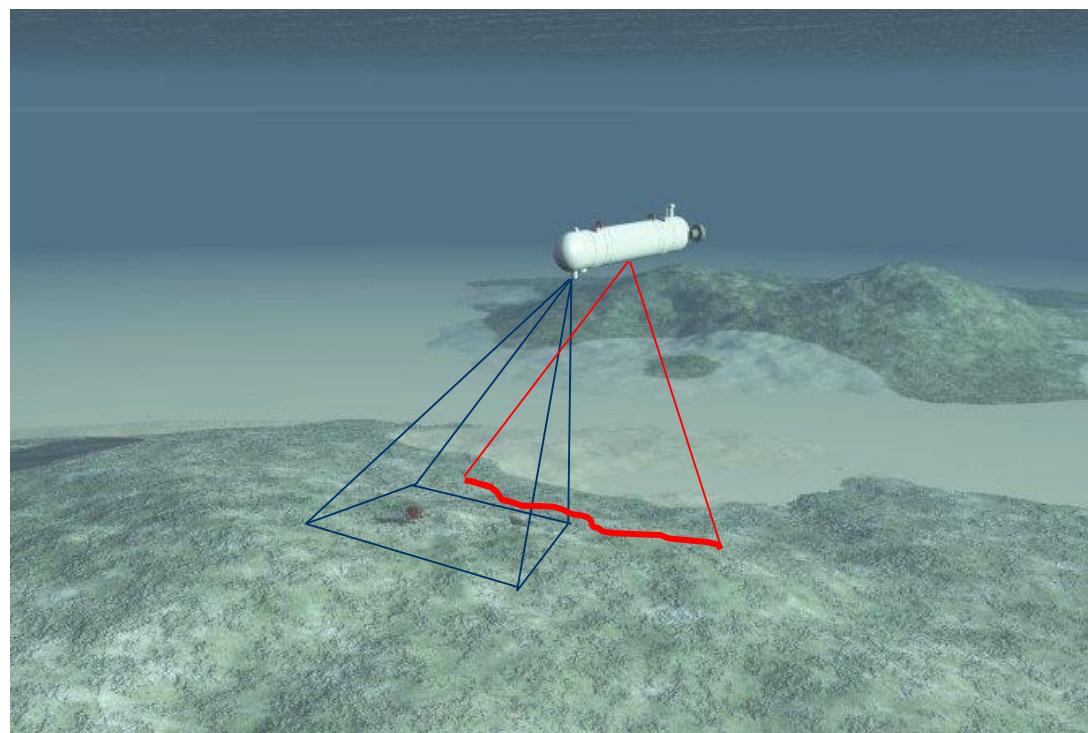
Terrain based navigation

What can I see?

Depth profile



Image

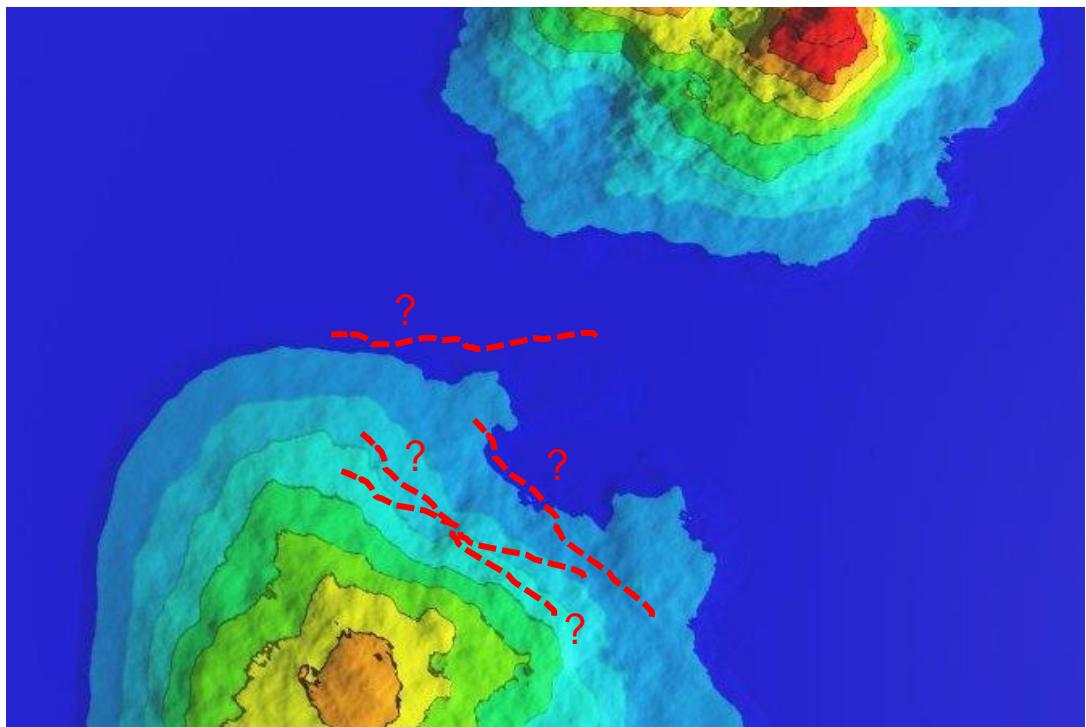




Terrain based navigation

Do I have a map?

Where can I be and see
this?



and if I have an image?

How do I find the
possible location?



How to find it???



Terrain based navigation

What I should observe here?



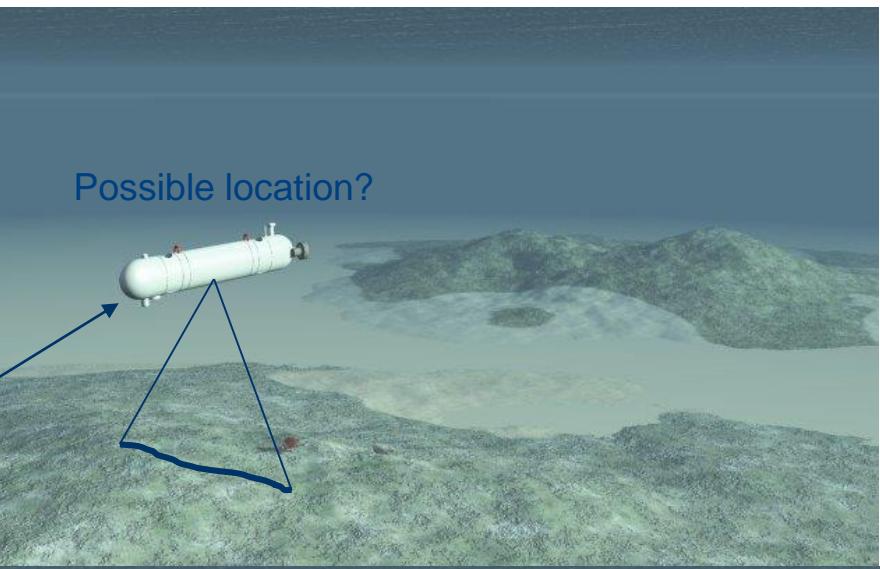
What I have seen?



Do they agree?



NO – I am not here



What I should observe here?



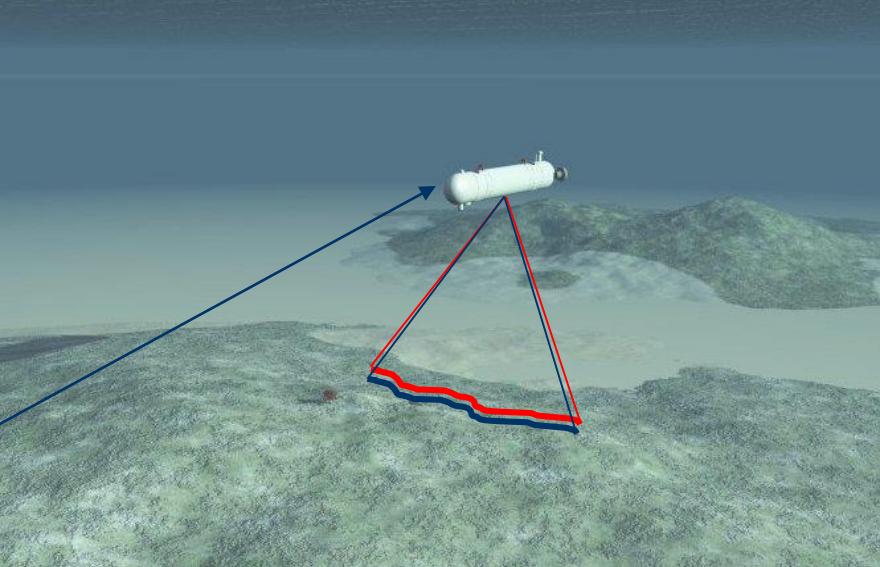
What I have seen?



Do they agree?



YES – I am here

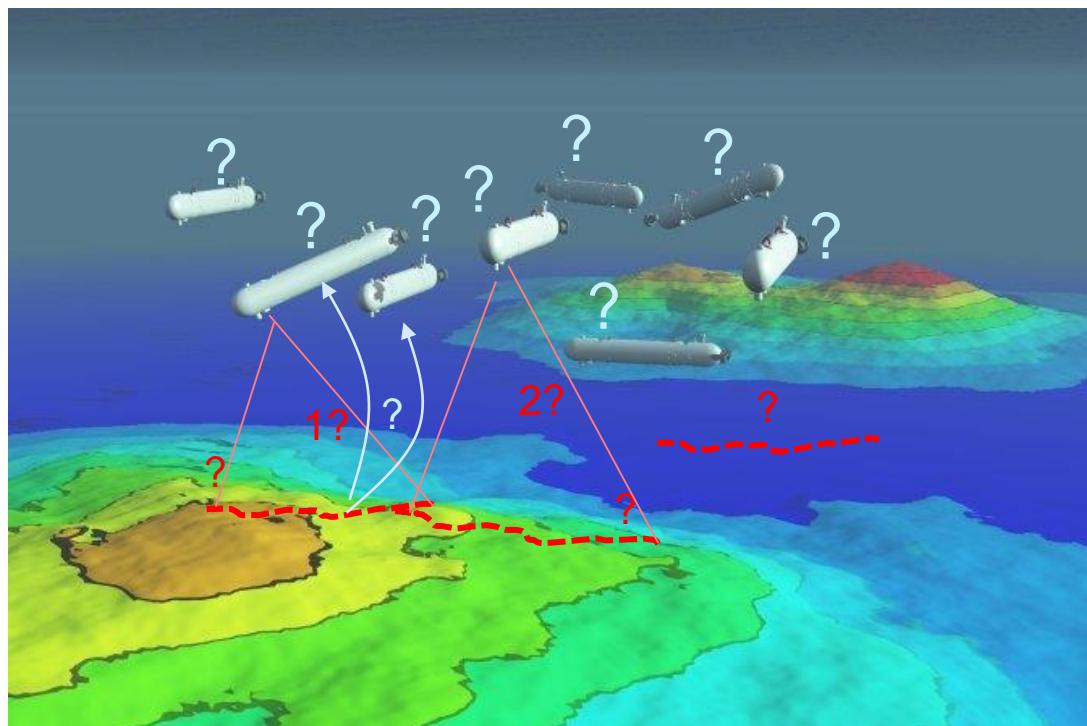
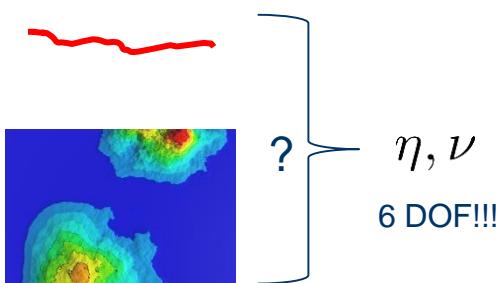




Terrain based navigation

Complete search in all the map can be problematic

- Large search space
- Computational power
- Multiple possible locations for the same observed data



Full unconstrained search is large, but usually some prior knowledge exists on initial location and current possibilities.

Start with initial position and continuously integrate measurement information to estimate location



Terrain based navigation

- Unstructured environments increase the complexity
- Problem in trying to match all the data to map
 - Dimension
 - Map usually does not have the same detail/granularity
 - Noise and uncertainty in the measurement process

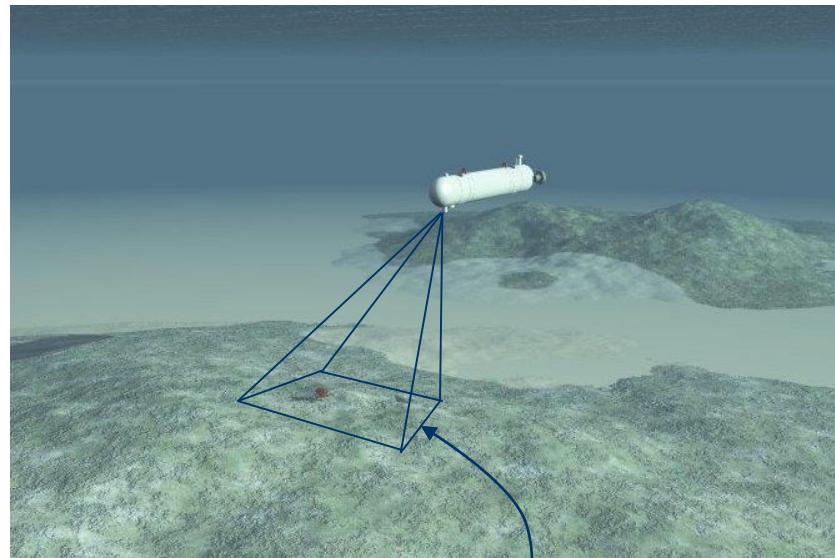


**Extract relevant information from the observations
Identify features or landmarks.**



Terrain based navigation

- Maps usually don't contain image information
- Even if a globally referenced ground truth image map (mosaic or other) exists
 - Matching difficult
 - Huge search space
 - Perspective, distortions, noise, etc

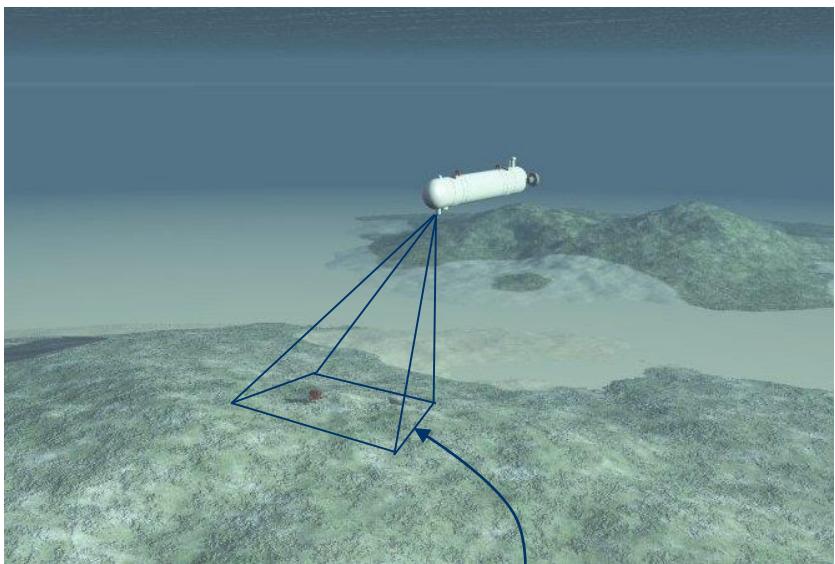


Difficult to know: “what image should I see here?”



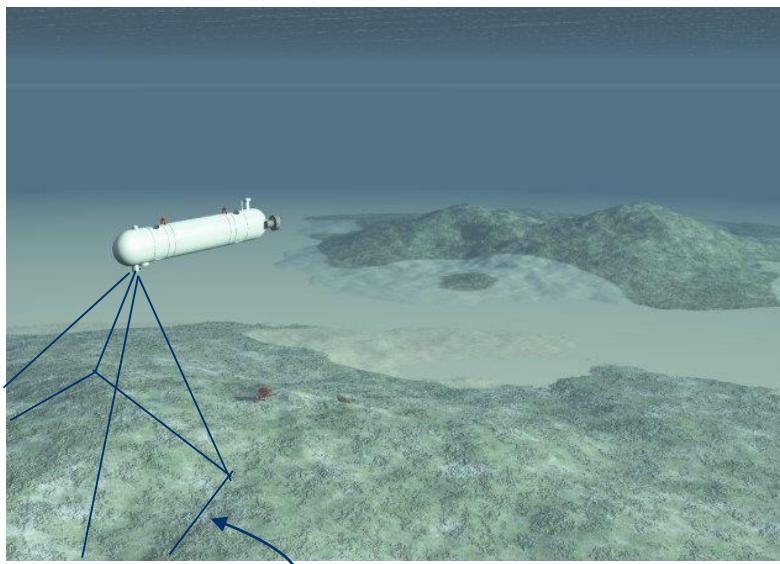


Terrain based navigation

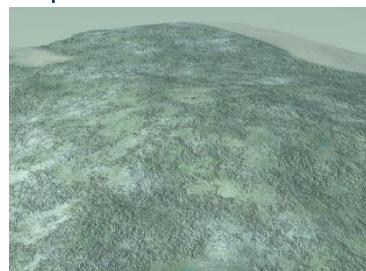


Where can I see this?

*This image seems better to answer
this question, why?*



Is there any relation?

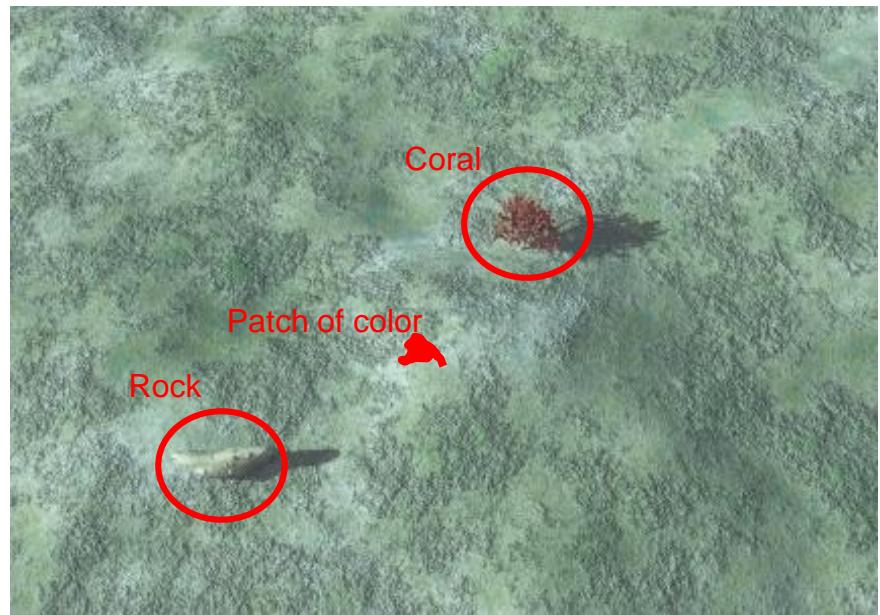


Where can I see this?

Terrain based navigation

Features

- Relevant, distinctive pieces of information
 - Can exist in map
 - Identification problems
 - Feature description
 - Similitude
 - Difficult to identify uniquely (possibly there also can be identical elements in the environment)
 - Unstructured environment increases difficulty
 - May not be static
 - Do not need to correspond to identifiable objects



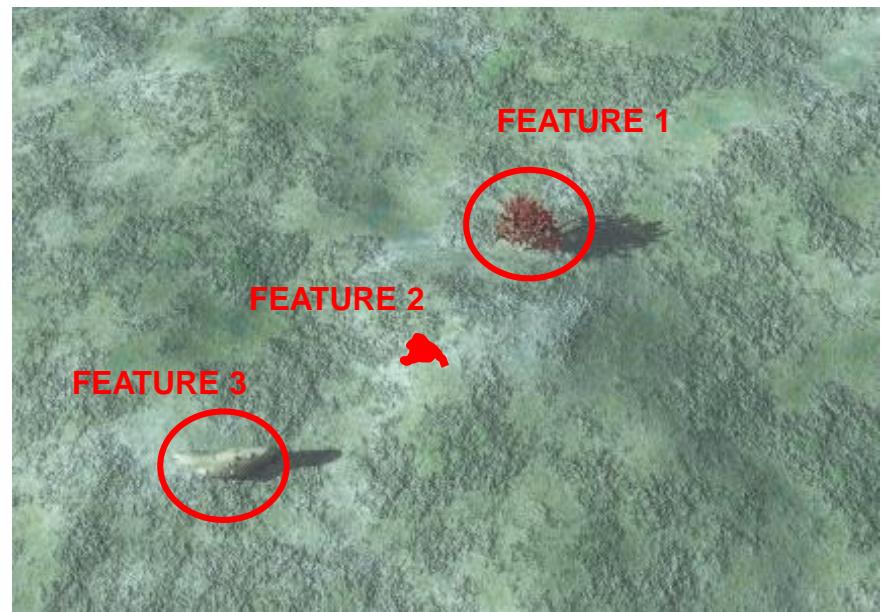
Feature points in the image
No special meaning, except for the identification process



Terrain based navigation

Identification

- Color
- Shape
- Template
- SIFTs, SURFs
- Other descriptors, etc



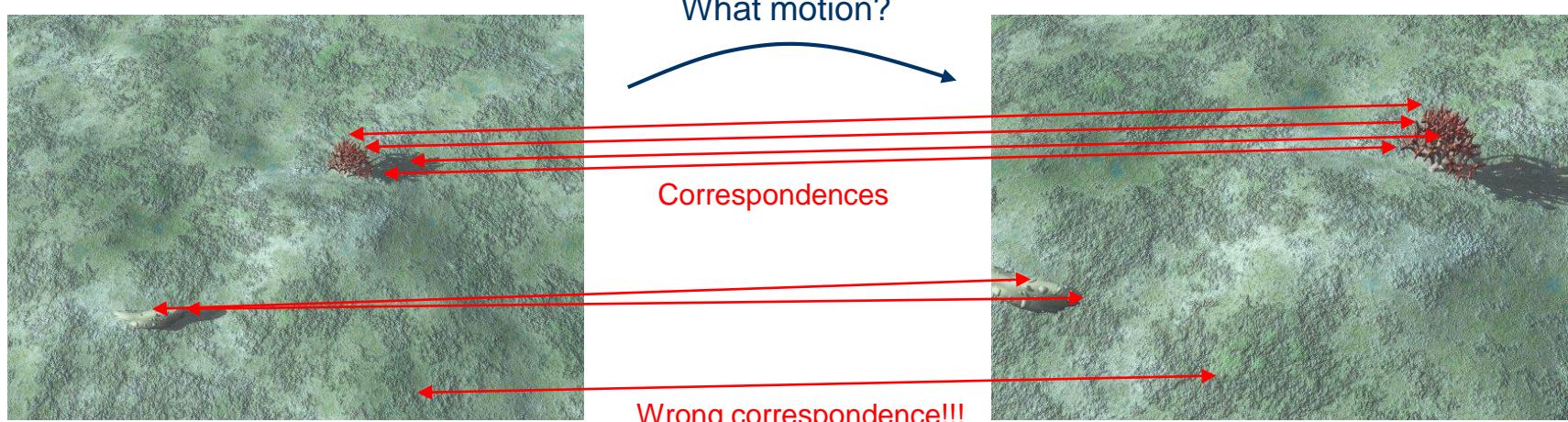
May or may not exist in the MAP



- Data association problem
- Map building

Visual odometry

- Sequence of images can be used to derive the motion of the observer between frames



Rotation R

Translation t

What about scale?

Velocity?

- If corresponding points between frames are available it is possible to obtain a rotation and translation between both, minus a scale factor
- Velocity can be derived if the time of frame is available

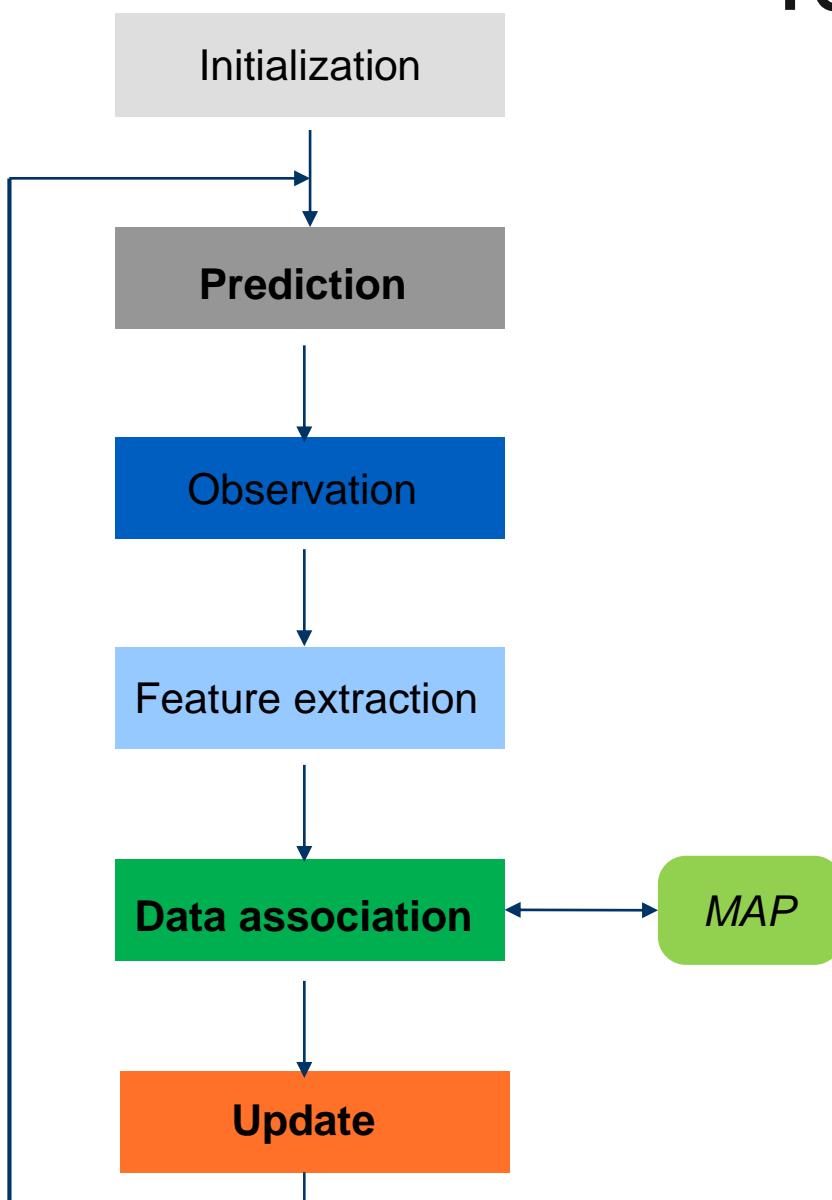


Visual odometry

- Used to provide relative position measurements – velocity
- Dependent on texture
- Difficult to use underwater due to imaging environment restrictions (loss of visibility, blur, noise...)
- For deep sea requires illumination – energy issues



Terrain based navigation



- Continuous process maintaining an estimate of the vehicle state
- Data association plays relevant role

SLAM – Simultaneous Localization and Mapping

What if there is no map? → SLAM Problem

Simultaneous Localization And Mapping (SLAM)

Build at the same time a map of the environment and locate itself in it

- Identify possible landmarks from the observations
- Maintain relative position in relation to a set of landmarks
- Perform the predict-update cycle on the landmark positions and vehicle positions

Landmark observation is correlated with vehicle position

Re-observation of landmarks required

SLAM Problem

Initial Conditions:

- Robot was no previous information about its localization
- Robot was no previous information about the environment

Working Principle:

- As the robot moves around:
 - Builds a map according to sensor gathered information (Map is relative to the estimated localization)
 - Localization is estimated by combining dead reckoning with map based observations
- Loop Closure
 - When the robot returns to a previously visited area:
 - Accumulated localization uncertainty can be reduced
 - Map and localization are correlated, so a correction in localization is back propagated to improve the map accuracy.

Useful link for SLAM references, algorithms and code:
www.openslam.org



SLAM Problem

- Simultaneous estimate of landmark and robot locations

\mathbf{x}_k State of the vehicle

\mathbf{u}_k Control at time $k-1$ to drive the vehicle to \mathbf{x}_k

\mathbf{m}_i Location of i th landmark

\mathbf{z}_{ik} Location of i th landmark

History of vehicle localization

$X_{0:k} = \{x_0, x_1, \dots, x_k\} = \{x_{0:k-1}, x_k\}$

Controls history

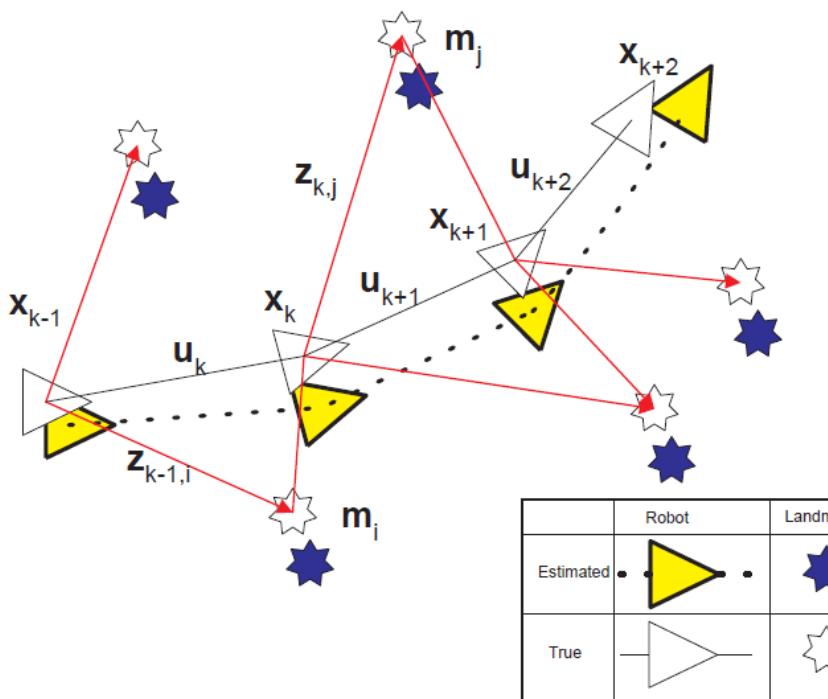
$U_{0:k} = \{u_1, u_2, \dots, u_k\} = \{U_{0:k-1}, u_k\}$

Landmarks

$\mathbf{m} = \{m_1, m_2, \dots, m_n\}$

All landmark observations

$Z_{0:k} = \{z_1, z_2, \dots, z_k\} = \{z_{0:k-1}, z_k\}$



[1] H. Durrant-Whyte and T. Bayley, "Simultaneous Localisation and Mapping (SLAM): Part I The Essential Algorithms", IEEE Robotics & Automation Magazine, 2006



Joint probability (posterior) of landmark localization given vehicle state

$$P(x_k, m | Z_{0:k}, U_{0:k}, x_0)$$

Observing model (observing probability given the landmark and vehicle state)

$$P(z_k | x_k, m)$$

Motion model as a state transition probability distribution

$$P(x_k | x_{k-1}, u_k)$$

- SLAM implemented in a classical predict-update cycle

Predict

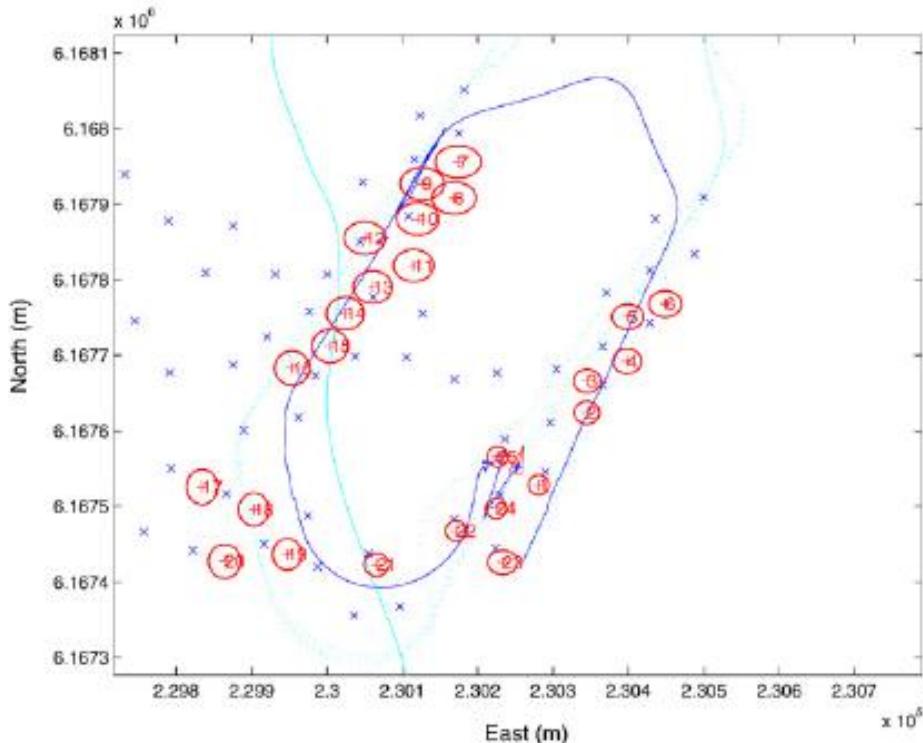
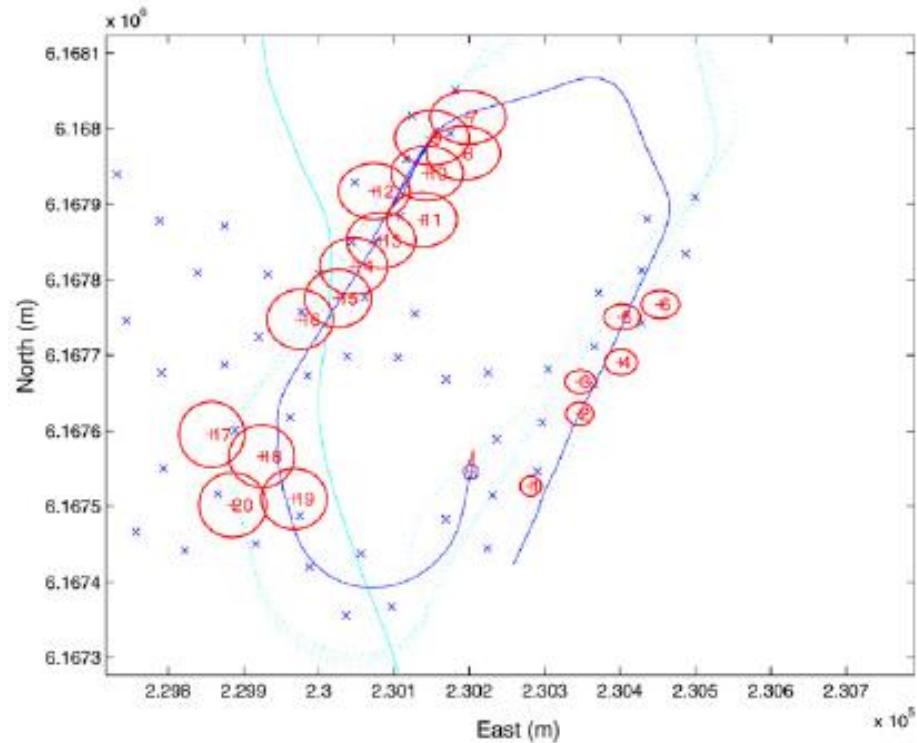
$$P(x_k, m | Z_{0:k-1}, U_{0:k}, x_0) = \int P(x_k | x_{k-1}, u_k) \times P(x_{k-1}, m | Z_{0:k-1}, U_{0:k-1}, x_0) dx_{k-1}$$

Update

$$P(x_k, m | Z_{0:k}, U_{0:k}, x_0) = \frac{P(z_k | x_k, m) P(x_k, m | Z_{0:k-1}, U_{0:k}, x_0)}{P(z_k | Z_{0:k-1}, U_{0:k})}$$



Loop Closure



“If loop closure fails, previously visited landmarks are re-mapped in the wrong global location... error accumulates without bounds and robot becomes lost” [2]

[1] Jonghyuk Kim and Salah Sukkarieh, “Real-time implementation of airborne inertial-SLAM”, Journal of Robotics and Autonomous Systems, 2007
[2] Newman, P.; Kin Ho, "SLAM-Loop Closing with Visually Salient Features," in Robotics and Automation, ICRA, pp.635-642, 18-22 April 2005



SLAM Frameworks

Extended Kalman Filter (EKF)

- Localization and map are represented by Gaussian probability distributions
- Noise should be white and Gaussian
- State vector is augmented to represent not only the robot pose but also map related states.
- SLAM follows a prediction/update cycle
 - **Prediction:** Robot localization is updated using a motion model
 - **Update:** Landmarks are observed:
 - New landmarks are added to the map
 - Existing landmarks are used to compute corrections
- Complexity grows quadratically with the number of states ^[1] – handle hundreds of landmarks

[1] Hugh Durrant-Whyte and Tim Bailey. Simultaneous Localization and Mapping (SLAM): Part I The Essential Algorithms. *IEEE Robotics and Automation Magazine*, 2, 200

EKF SLAM

- EKF complexity grows quadratically with the number of states [1]
 - Strategies for removing high percentage of landmarks, maintaining only meaningful ones, without loss in map consistency [1].
 - SLAM is performed in a local area around robot localization [2]. Estimates are later combined with the global map when the robot moves away.
 - Map partitioning in low correlated sub-regions [3].

[1] Gamin Dissanayake, Hugh F. Durrant-Whyte and Tim Bailey, ``A Computationally Efficient Solution to the Simultaneous Localisation and Map Building (SLAM) Problem''. ICRA, pages 1009--1014, 2000.

[2] Jose Guivant and Eduardo Nebot, "Optimization of the simultaneous localization and map-building algorithm for real-time implementation," Robotics and Automation, IEEE Transactions on , vol.17, no.3, pp.242,257, Jun 2001.

[3] J.J. Leonard and H.J.S. Feder, ``A computational efficient method for large-scale concurrent mapping and localisation''. In Proceedings of the Ninth International Symposium on Robotics Research, pages 169–176, 2000

Particle filters

- Recursive Bayesian estimator
- Non-parametric state representation by means of particles, composed by samples and weights.
- Do not impose any specific distribution shape
- Handle high non-linear and multi-modal distributions
- For a infinite number of particles, the posterior density function approximates the optimal estimate [9].
- Complexity in traditional Particle Filters grows proportionally with the number of particles – even for small maps, SLAM implementations are not feasible

[1] M. Sanjeev Arulampalam, Simon Maskell, and Neil Gordon, ``A Tutorial on Particle Filters for Online Nonlinear/non-Gaussian Bayesian Tracking''. *IEEE Transaction on Signal Processing*, 50:174–188, 2002

Fast SLAM^[1]

- Exploits conditional independence of the landmarks given the robot trajectory
- SLAM problem is decoupled into a localization problem and a set of landmark estimation problems, conditioned on the localization
- Localization is estimated through a particle filter.
 - Each particle contains its own map.
 - Inside a particle, each landmark is estimated in a dedicated EKF.
- Advantages:
 - Low complexity $O(P \cdot \log M)$ – Handles thousands of landmarks
 - Each particle develops its own data association

[1] Michael Montemerlo, Sebastian Thrun, Daphne Koller, and Ben Wegbreit. “FastSLAM: A Factored Solution to the Simultaneous Localization and Mapping Problem.” In Proceedings of the AAAI National Conference on Artificial Intelligence, pages 593–598. AAAI, 2002

Graph SLAM^[1]

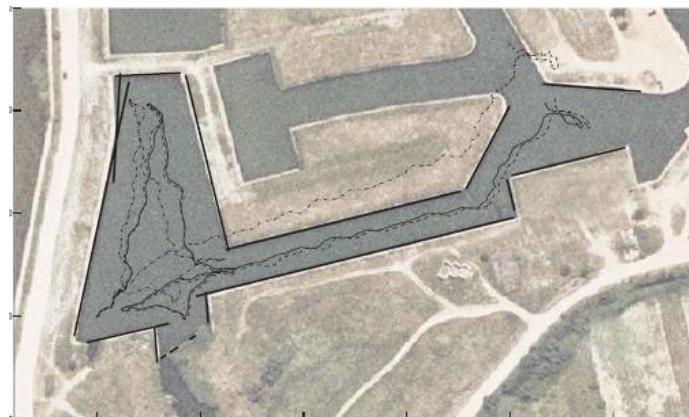
- SLAM formulation using a graph:
 - Nodes correspond to robot poses
 - Edges represent sensor measurements and establish constraints between nodes
 - An optimization step finds the node configuration that maximizes the measurement consistency

[1] Grisetti, G.; Kümmerle, R.; Stachniss, C.; Burgard, W., "A Tutorial on Graph-Based SLAM," in Intelligent Transportation Systems Magazine, IEEE , vol.2, no.4, pp.31-43, 2010



Some Underwater SLAM implementations

- SLAM in a Marina Environment
 - EKF SLAM for structured environments.
 - Prediction follows a constant velocity kinematic model
 - State updates:
 - DVL – Bottom lock 1.5Hz velocity updates plus depth measurement
 - Compass+inclinometer – Low cost motion reference unit provides attitude updates at 0.1Hz.
 - Landmarks from MSIS
 - No Loop closure
 - Accuracy around 15 meters for a trajectory of 600 meters.
 - Structured environment, since straight lines are extracted from MSIS data.



Dash-dot – slam map
Dashed - GPS
Solid – slam traj



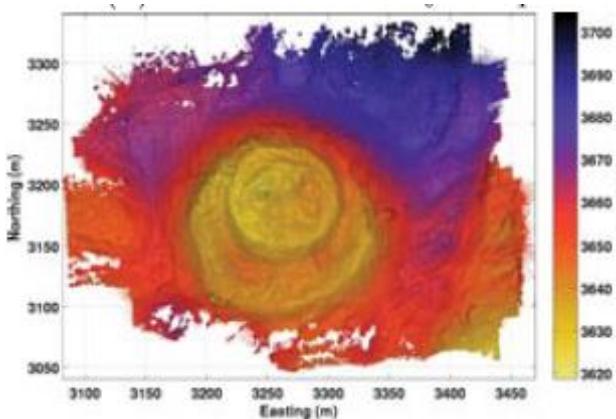
Images from [1]

[1] D. Ribas et al, Underwater SLAM in Man-Made Structured Environments“, Journal of Field Robotics, Wiley, 2008



BPSLAM

- Bathymetry based SLAM
- SLAM with particle filter
- Featureless - No data association
- Discretized bathymetry map (evidence grid) – 2.5 D
 - Single depth estimate per cell
 - Updated with EIF (Extended Information)
- Vehicle states estimated in separated EKF
- Adaptive particle sizing



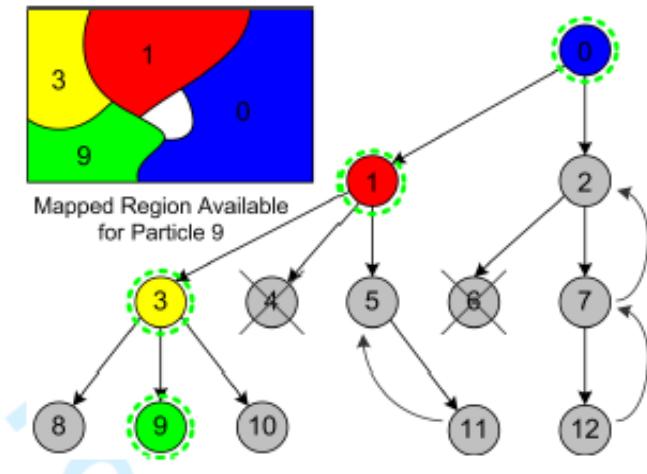
[1] S. Barkby, S. B. Williams, O. Pizarro, and M. V. Jakuba, "A featureless approach to efficient bathymetric slam using distributed particle mapping." *Journal of Fields Robotics*, vol. 28, no. 1, pp. 19–39, 2011

BPSLAM

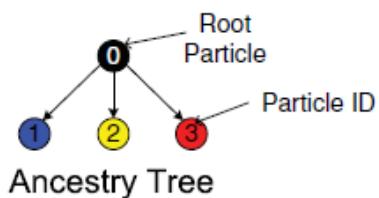
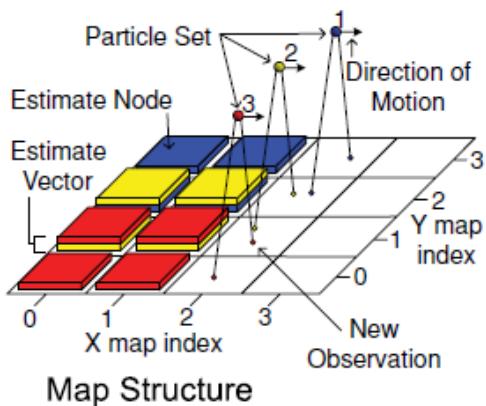
- Hierarchical
- Each resample particle does not copy the map but only points to it
- Each particle has only a map (its own grid) with observations after resampling
- Full map obtained by following the tree

Marked nodes indicate map for particle 9
Leafs current particles

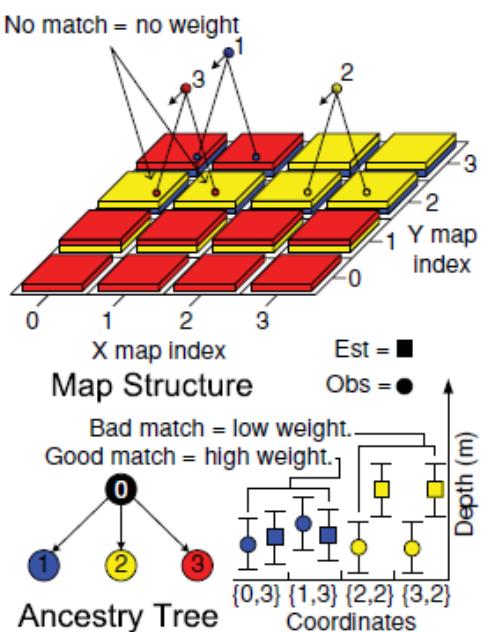
Figure from [1]



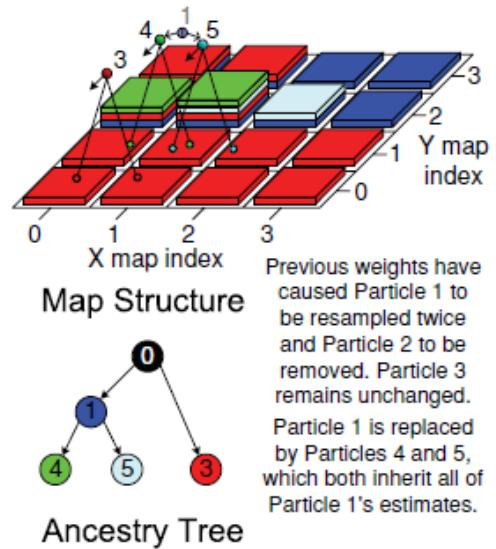
[1] S. Barkby, S. B. Williams, O. Pizarro, and M. V. Jakuba, "A featureless approach to efficient bathymetric slam using distributed particle mapping." *Journal of Fields Robotics*, vol. 28, no. 1, pp. 19–39



(a) Particles forming initial depth estimates



(b) Particle weighting procedure upon reobserving seabed



(c) Particle resampling that results from weighting

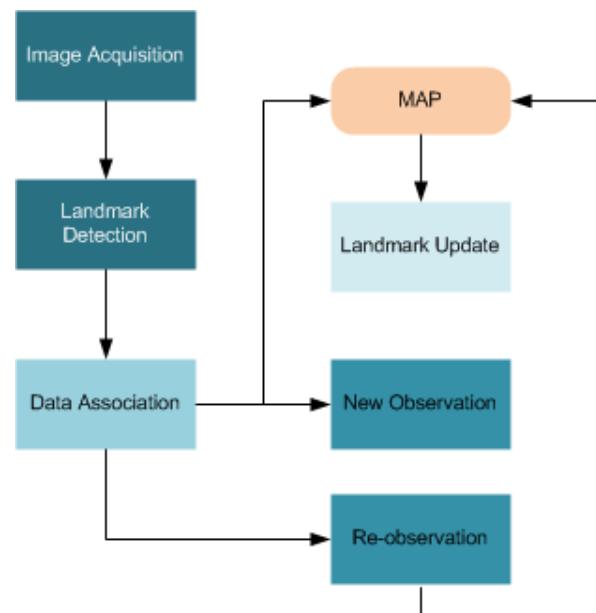
Figure from [1]

[1] S. Barkby, S. B. Williams, O. Pizarro, and M. V. Jakuba, "A featureless approach to efficient bathymetric slam using distributed particle mapping." *Journal of Fields Robotics*, vol. 28, no. 1, pp. 19–39, 2011



Real Time Visual SLAM

- Relative navigation for ROVs without additional sensors
 - Deep sea ROV navigation for legacy systems
 - Additional functionalities
- Navigation aid for AUVs near bottom or close to structures
 - For ex: for precise landing in benthic transporting robots
- Visual navigation in multiple robot operations at close range (docking, motion coordination etc)



RT-SLAM framework

- Real-Time SLAM software infrastructure
- Developed at LAAS-CNRS
- Monocular vision based
- FastSLAM
- Landmark parameterization
 - Inverse depth
 - Re-parameterization of converged landmarks to euclidean point (3 params vs 6, leading to computation and memory gains)
- Image processing
 - Harris corner detector
 - ZNCC for point matching
- Constant velocity prediction model
- Active search – One-point RANSAC

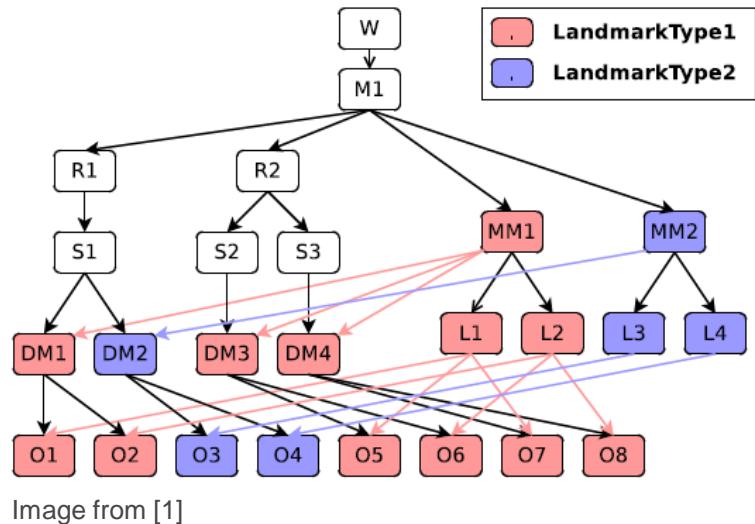


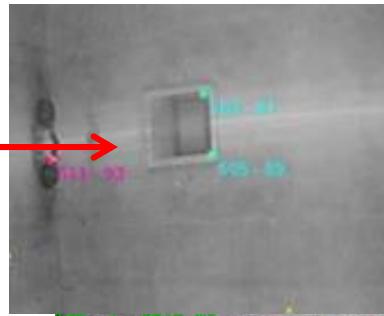
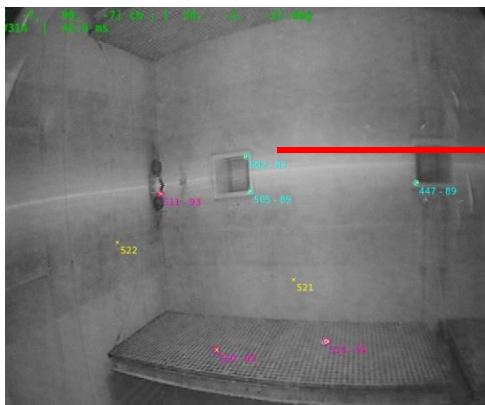
Image from [1]

W – world
M – map
R – robot
S – sensor
O – observation
MM – map manager
DM – data manager

[1] C. Roussillon et al., "RT-SLAM: A Generic and Real-Time Visual SLAM Implementation," in *Computer Vision Systems*, vol. 6962, J. L. Crowley, B. A. Draper, and M. Thonnat, Eds. Berlin, Heidelberg: Springer Berlin Heidelberg, 2011, pp. 31–40



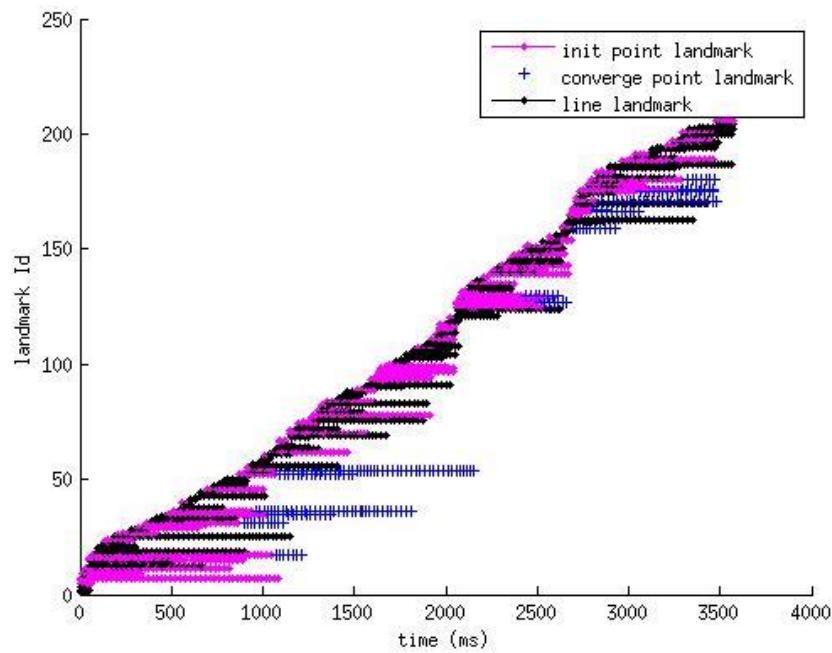
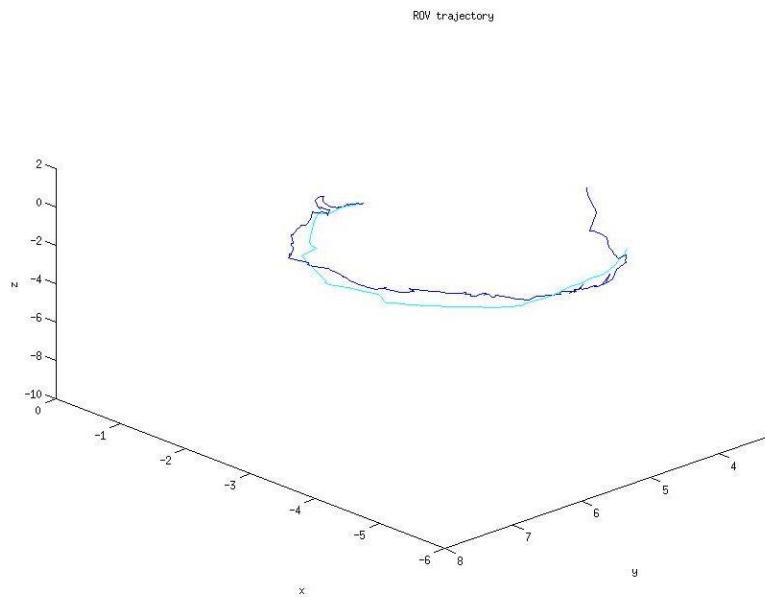
Point and line features detection



- Motion dynamic changes lead to divergence (rotations, lack of INS)
- Dependence on the low quality image
- Low number of features in the scenario



ROV trajectory and landmark evolution

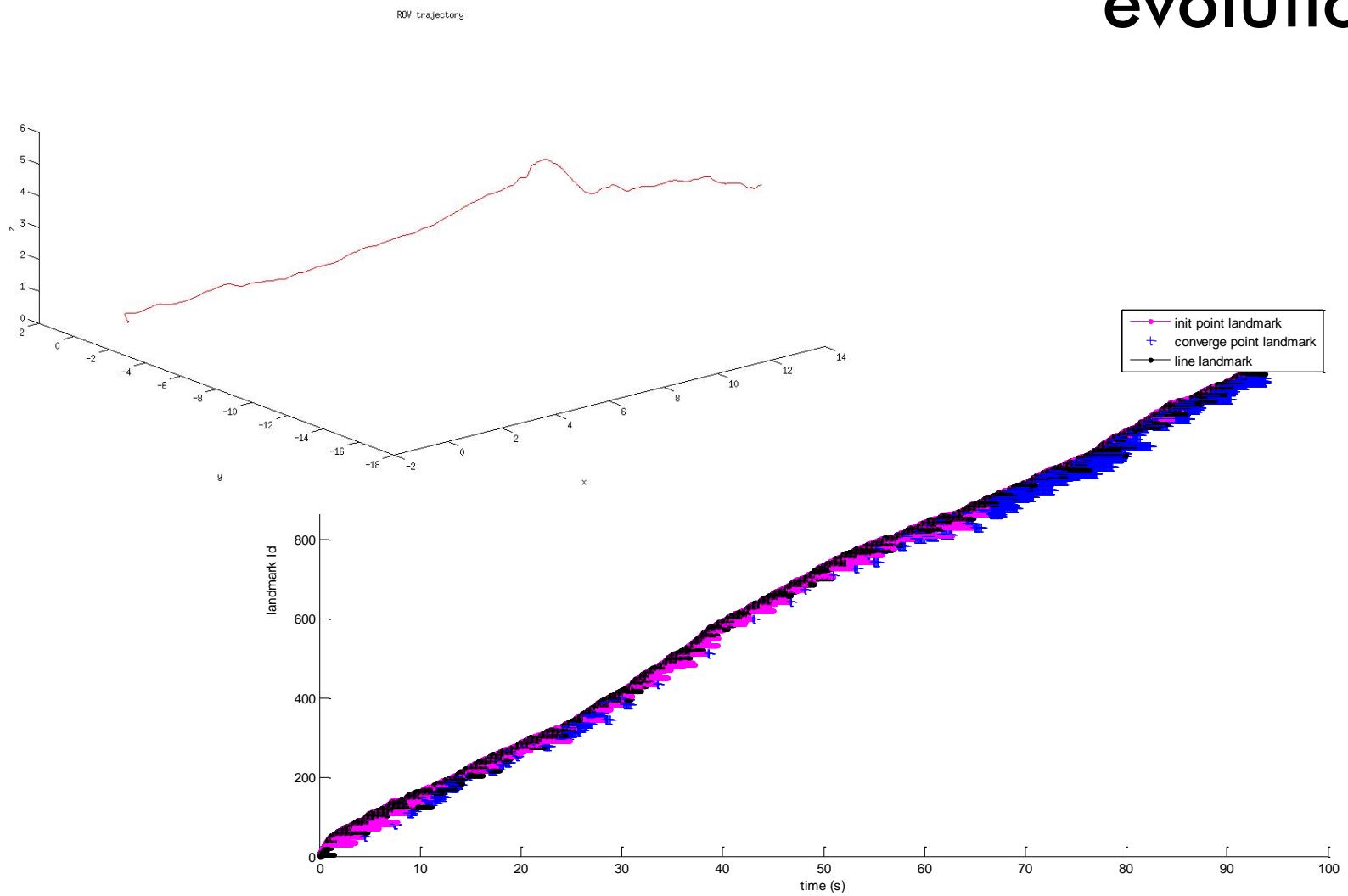


Deep sea

- Only monocular vision
 - Feature extraction even with harsh conditions
 - Useful for when only image is available

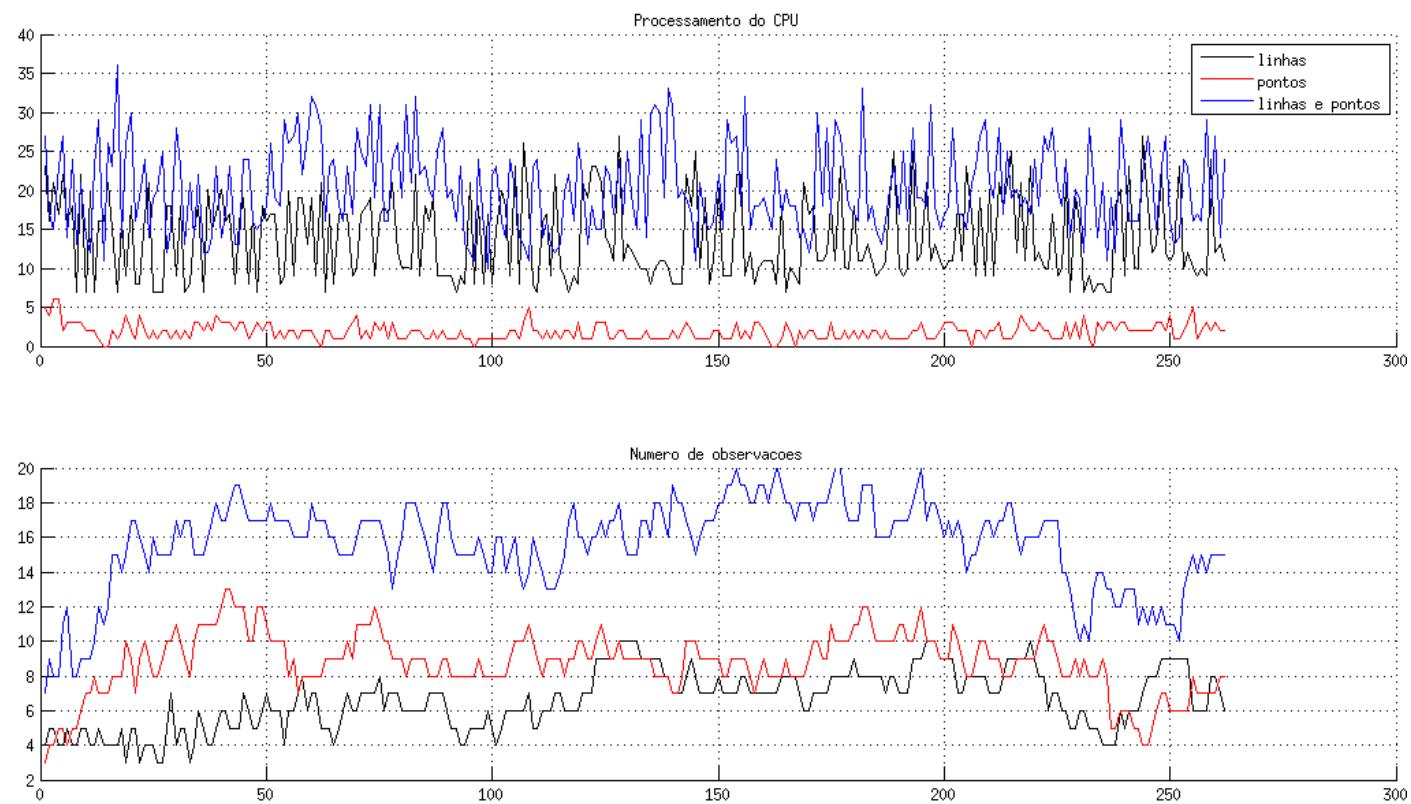


Trajectory and landmark evolution





Realtime performance



- Monocular up to 30 fps – Pentium Dual core
- For the test tank case real time performance with an Intel Atom Dual Core (25 W)



Underwater SLAM implementations

Author	Scenario	Main Sensors	Estimator	Landmarks	Loop closure	Submapping
[1], 2006	Sea coast	DVL, INS, Compass, LBL	EKF	Beacon Locations	No	No
[2], 2007	Flooded caverns	DVL, INS, Depth sensor, Profiling sonar	PF	Walls	No	No
[3], 2008	Shallow waters	DVL, Compass, Tilt sensor, Pressure sensor, Stereo vision	EIF	SURF	Yes	No
[4], 2008	Marina	DVL, MRU, MSIS	EKF	Walls	Yes	Yes
[5], 2009	Shallow waters	DVL, Heading Gyroscope, FLS	2D EKF	FLS	No	No

[1] E. Olson, J. Leonard, and S. Teller, "Robust range-only beacon localization," *IEEE Journal of Oceanic Engineering*, vol. 31, no. 4, pp. 949–958, 2006.

[2] N. Fairfield, D. Jonak, G. Kantor, and D. Wettergreen, "Field results of the control, navigation, and mapping systems of a hovering auv," in *UUST*, 2007

[3] I. Mahon, S. Williams, O. Pizarro, and M. Johnson-Roberson, "Efficient view-based slam using visual loop closures," *IEEE Transactions on Robotics*, vol. 24, no. 5, pp. 1002–1014, 2008.

[4] D. Ribas, P. Ridao, J. Tardos, and J. Neira, "Underwater slam in manmade structured environments," *Journal of Field Robotics*, vol. 25, pp. 898–921, 2008.

[5] Koh, A.C.T.; Wijesoma, W.S.; Pua, S.L.; Lee, K.W.; Kalyan, B., "Shallow waters SLAM experiments on meredith AUV using forward looking sonar," in *OCEANS*, pp.1-6, 26-29 Oct. 2009



Underwater SLAM implementations

Author	Scenario	Main Sensors	Estimator	Landmarks	Loop closure	Submapping
[18], 2009	Sea	DVL, Gyrocompass, GPS, Depth sensor, SSS	Graph SLAM	SSS Landmarks	No	No
[19], 2011	Sea	Camera	PF	SIFT	No	No
[20], 2010	Sea	Sonar, Multibeam, DVL, Compass	PF	Terrain	Yes	Yes
[21], 2010	12 m depth tests	DVL, IMU, Depth sensor, SSS	EKF	SSS Landmarks	Yes	Yes
[22], 2011	Shallow waters	DVL, IMU, Camera	EKF	SURF	Yes	Yes

[6] Luc Jaulin, "A Nonlinear Set-membership Approach for the Localization and Map Building of an Underwater Robot using Interval Constraint Propagation", IEEE Transaction on Robotics, vol 25, no. 1, pp. 88–98, 2009

[7] S. Augenstein and S. Rock, "Improved frame-to-frame pose tracking during vision-only slam/sfm with a tumbling target," in ICRA, pp. 3131–3138, 2011

[8] S. Barkby, S. B. Williams, O. Pizarro, and M. V. Jakuba, "A featureless approach to efficient bathymetric slam using distributed particle mapping." Journal of Fields Robotics, vol. 28, no. 1, pp. 19–39.

[9] Aulinas, J.; Llado, X.; Salvi, J.; Petillot, Y.R., "Selective Submap Joining for underwater large scale 6-DOF SLAM," in Intelligent Robots and Systems (IROS), 2010 IEEE/RSJ International Conference on , vol., no., pp.2552-2557, 18-22 Oct. 2010

[10] Aulinas, J.; Carreras, M.; Llado, X.; Salvi, J.; Garcia, R.; Prados, R.; Petillot, Y.R., "Feature extraction for underwater visual SLAM," in OCEANS, 2011 IEEE - Spain , vol., no., pp.1-7, 6-9 June 2011