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# *Design of fuzzy systems*

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## Steps for fuzzy system design

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- Problem definition
- Parametrization of the model: concepts
- Mapping definition: rules
- Implementation
- Testing

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## Problem definition

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Analogous to what done in classical model design

- Selection of the **input** variables
- Selection of the **output** variables
- Selection of the **goals** of the model

## Selection of input variables

In principle, numerical or ordinal variables so that it is possible to define fuzzy sets

Variables that can be

- “perceived” from **sensors**, **data** or **users**
- computed from perceived variables (error, derivatives, composition of variables)

In general, there are no a priori preclusions to select variables: it depends on the problem and the designer sensibility

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## Selection of output variables

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Output variables are the results of the model, so come directly from the modeler needs.

For instance:

- for a control application, we might either have the control variables, or the respective increments
- for a decision support system we might have the decision to be taken

## Selection of the goals of the fuzzy model

The goals of the fuzzy models depend on the specifications

For instance, a control system may reach the set point as soon as possible, with the least overshooting, as robustly as possible, etc.

The goals should always be stated in advance, and guide the design.

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## System parametrization

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- Selection of membership functions for all variables
- Selection of the inferential mechanism (which T- norms...)
- Selection of eventual fuzzyfication and defuzzyfication

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## Membership function selection

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- Number of granules: not too many, nor too few (7?)
- Coverage: an output for any input value
- Boundary considerations
- How to reach equilibrium?
- Value density: more attention where needed
- Crosspoint: smoothness in the output
- Singletons or more general MFs for the output

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## How many MFs?

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It depends on the application.

In general from 3 to 7.

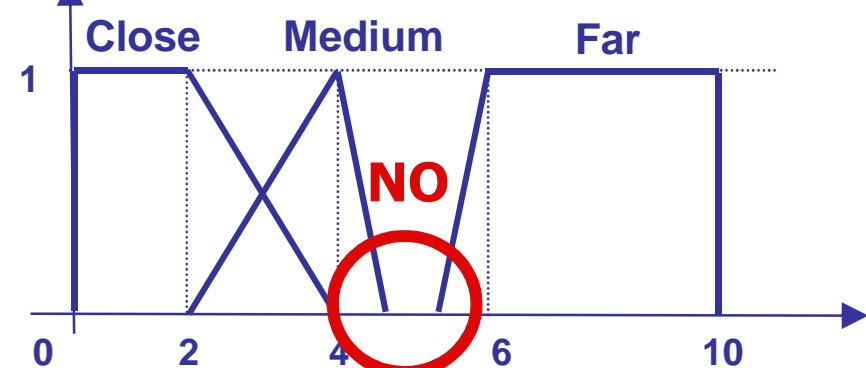
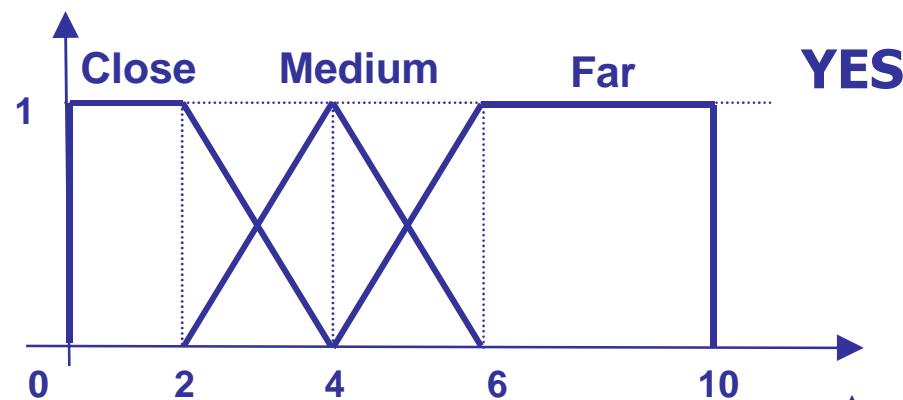
Why?

The “magic” number 7: people usually cannot manage more than 7  $\pm$  2 concepts at a time

Let’s try...

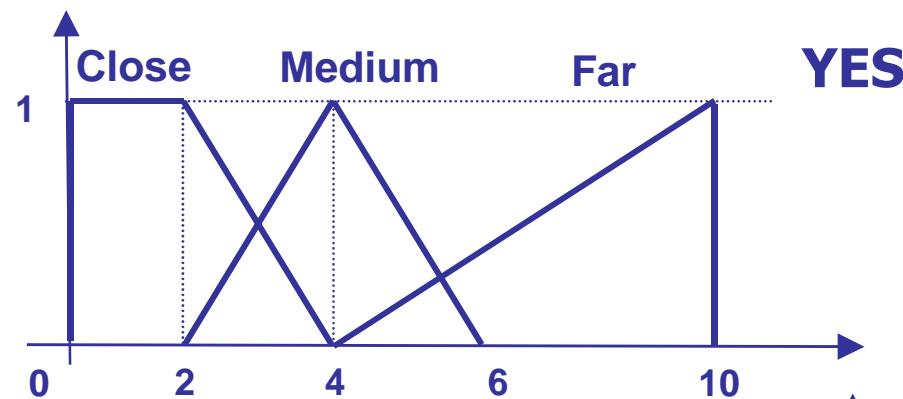
## An output for each input

Any point in the range of input variables has to be covered by at least one fuzzy set participating to at least one rule

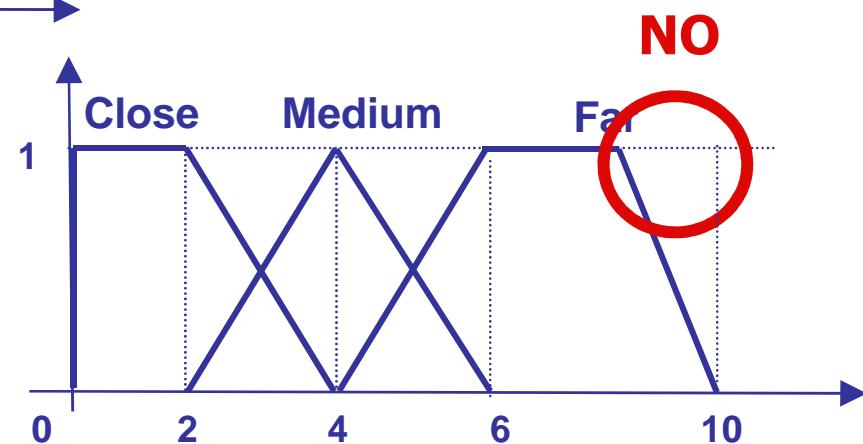


## Boundary considerations

Boundary should be covered with maximum value

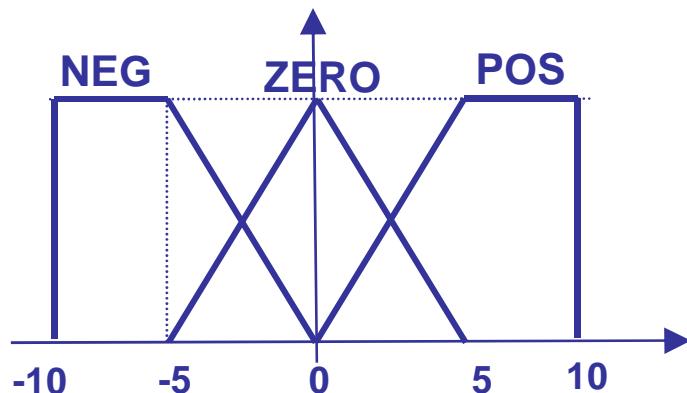


↑  
Saturation or not?



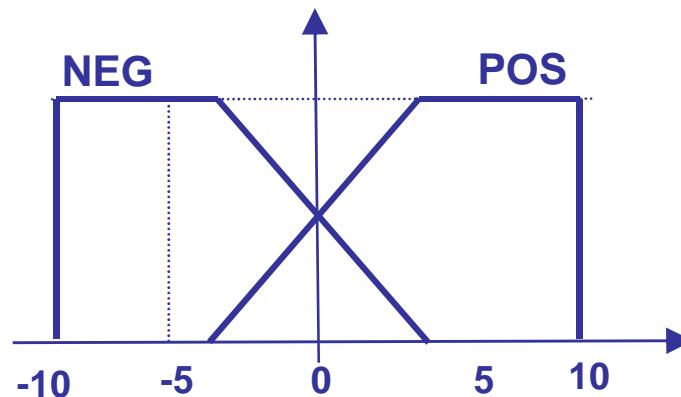
## How to reach equilibrium

### A mediating MF



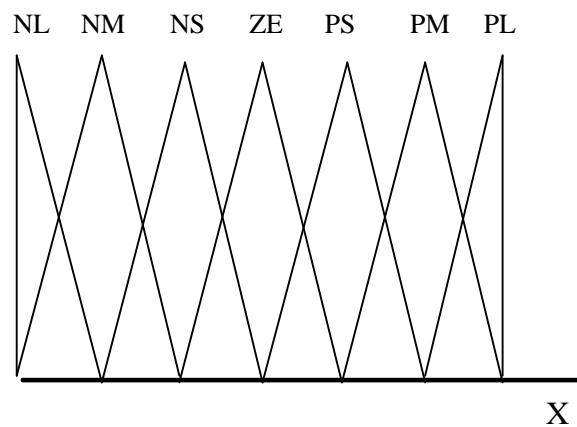
IF (X is POS) THEN (U is NEG)  
IF (X is NEG) THEN (U is POS)  
IF (X is ZERO) THEN (U is ZERO)

Just pushing...



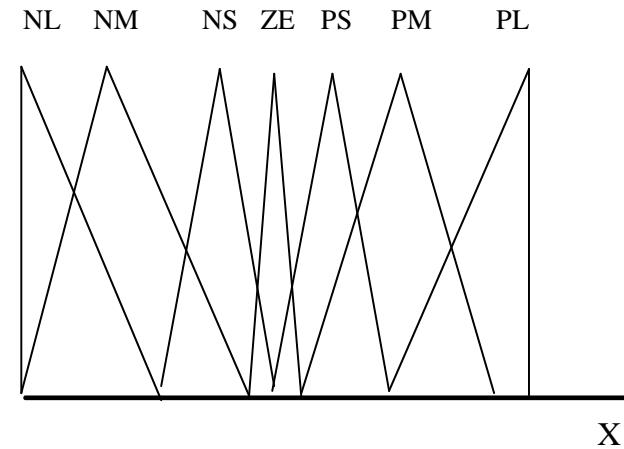
## How to distribute the MFs

Evenly distributed



Max robustness to noise

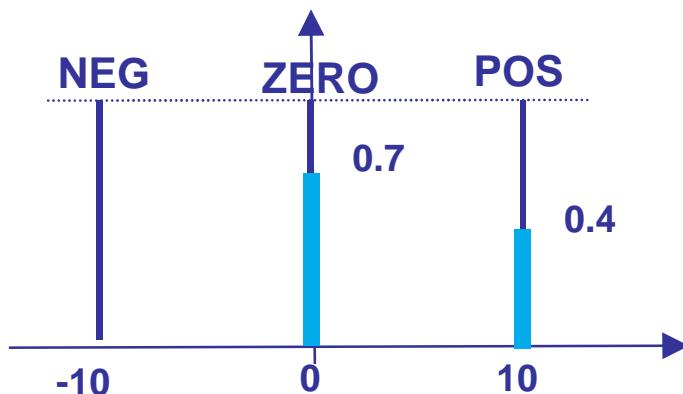
Unevenly distributed



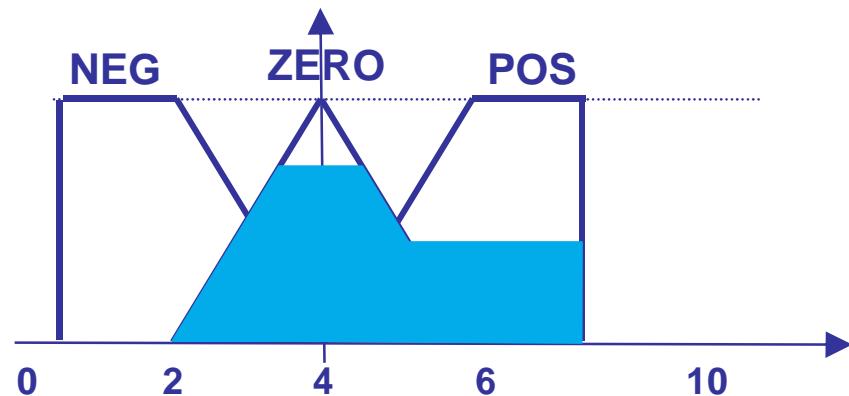
More precision where needed

## Singletons or other for the output

**Singletons: the weight of any cut is proportional to the cut level**



**Shapes: the relative weight is higher for lower cuts**



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## How are MFs defined?

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### Single expert

- Objective evaluation (e.g.: error wrt a set point)
- Interview (e.g.: operator of a control room)

### Multiple experts

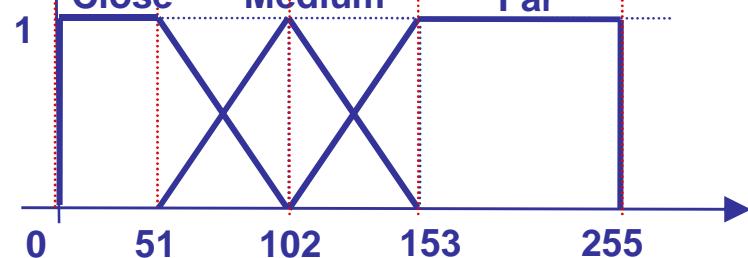
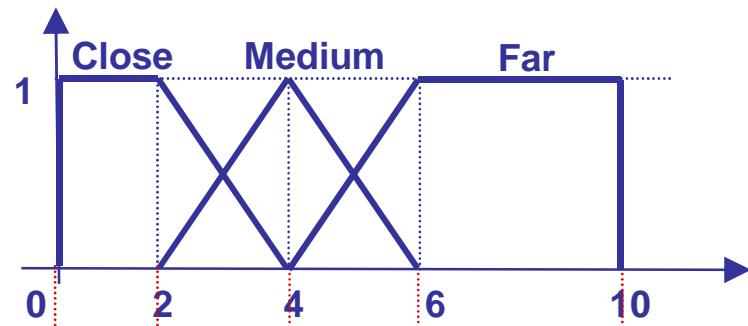
- Probabilistic elaboration, possibly weighted by the expert reliability

### Automatic systems (NN, GA, ...) working on data

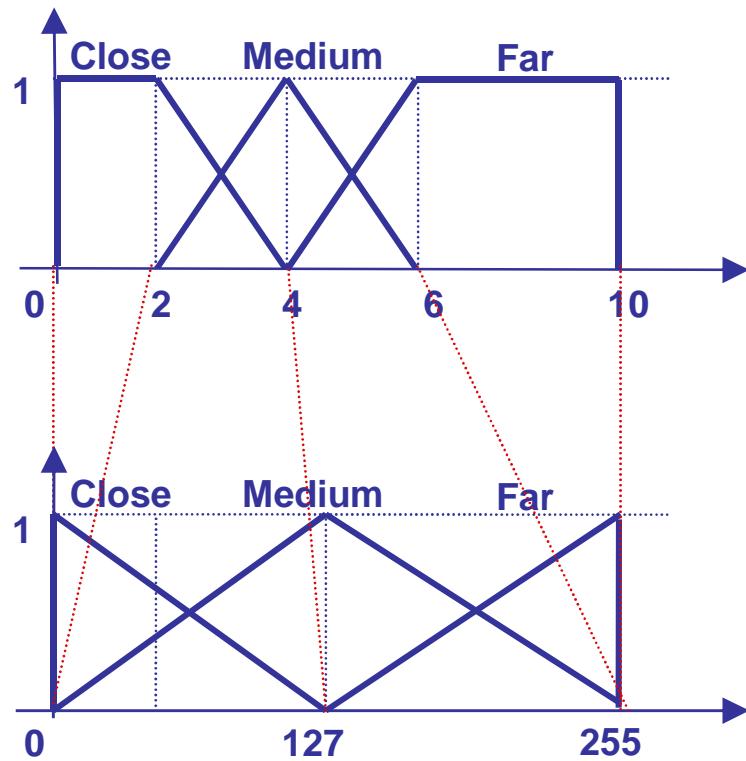
## Normalization

It is common to normalize data on a standard range (e.g., 0 – 255)

Linear normalization



NON-linear normalizaton

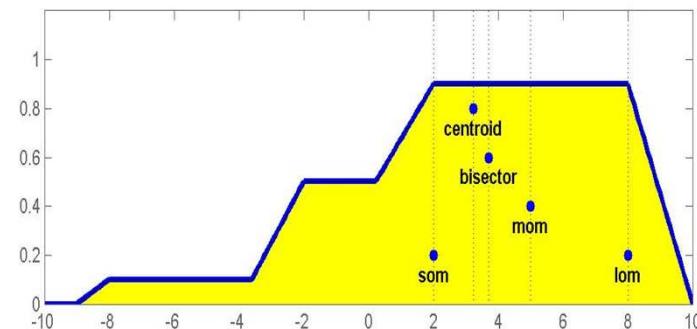
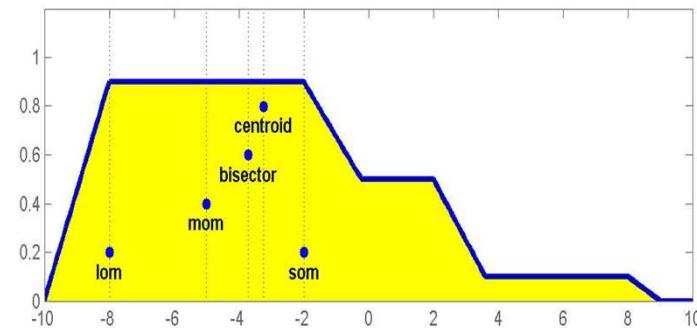


mainly used in fuzzy µcontrollers  
to exploit the available precision

## Defuzzyfication

Many different possibilities:

- Centroid
- Bisector
- Average of maxima
- Lowest maximum
- Highest maximum
- Center of the highest area
- ...



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## Rule definition

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From experience

- Introspective analysis
- Structured interview

From another model (e.g., a mathematical model)

By using machine learning, or self-tuning techniques (NN, GA, ...)

## Selection of inferential engine

The inferential engine depends on the operators selected for:

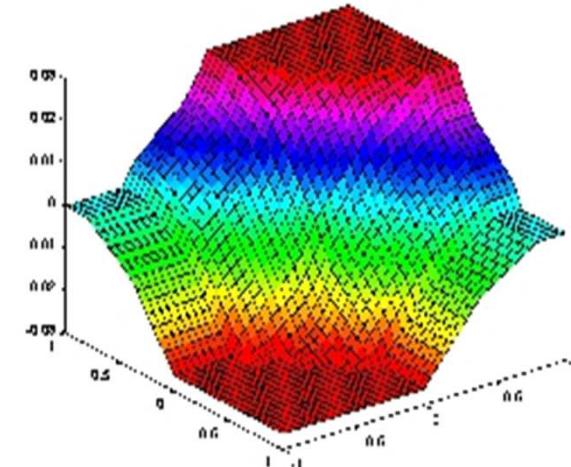
- AND of antecedent clauses
  - min: the worst degree of matching is the most relevant
  - product: all the degrees of matching are relevant
- **Detachment**: combination with the rule weight
  - min
  - product
- Aggregation of degrees of the same consequent
  - max: the best degree is the most relevant
  - probabilistic sum: all the collected knowledge is taken into account

## Testing

Aim: verify the design goals

Possible activity

- Dynamic simulation, if the model of the subject is available
  - E.g., if the model of the controlled system is available
- “Static” simulation (single I/O check, output (control) surface study)
- Test on the process, possibly under safe conditions



## Available development tools

Many tools, mainly for **fuzzy control** (including Matlab)

Common features:

- guided definition of rules and MF
- visualization of control surfaces
- suite of MFs, operators and defuzzification methods
- support to testing
- support to learning
- optimized code production, for many processors

Some tools for the development of **generic fuzzy rule systems**:  
FOOL, Fuzzy Clips, FLIP, FCL, ...

## FCL: an IEEE standard

IEEE-IEC document 61131-7

An example: model the aggressiveness of a character in a videogame

FUNCTION\_BLOCK

\\* VAR definition \*\

VAR\_INPUT

    Our\_Health    REAL; (\* RANGE(0 .. 100) \*)  
    Enemy\_Health  REAL; (\* RANGE(0 .. 100) \*)

END\_VAR

VAR\_OUTPUT

    Aggressiveness  REAL; (\* RANGE(0 .. 4) \*)

END\_VAR

## FCL Example (2): MF definition

```
\* MF definition *\
```

```
FUZZIFY Our_Health
```

```
TERM Near_Death := (0, 0) (0, 1) (50, 0) ;
```

```
TERM Good := (14, 0) (50, 1) (83, 0) ;
```

```
TERM Excellent := (50, 0) (100, 1) (100, 0) ;
```

```
END_FUZZIFY
```

```
FUZZIFY Enemy_Health
```

```
TERM Near_Death := (0, 0) (0, 1) (50, 0) ;
```

```
TERM Good := (14, 0) (50, 1) (83, 0) ;
```

```
TERM Excellent := (50, 0) (100, 1) (100, 0) ;
```

```
END_FUZZIFY
```

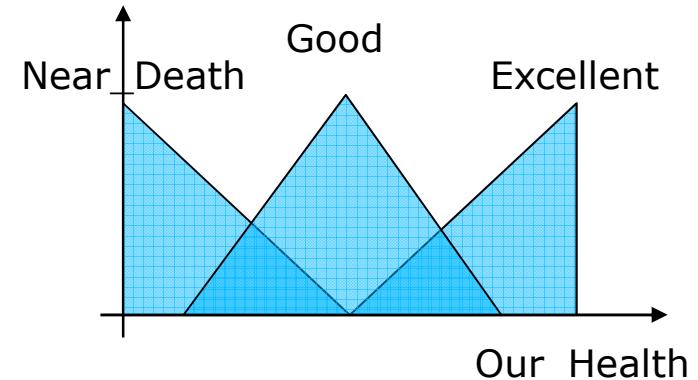
```
FUZZIFY Aggressiveness
```

```
TERM Run_Away := 1 ;
```

```
TERM Fight_Defensively := 2 ;
```

```
TERM All_Out_Attack := 3 ;
```

```
END_FUZZIFY
```



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## FCL Example (3): defuzzyfication

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\\* Definition of the defuzzyfication method \*\

DEFUZZIFY Aggressiveness

  METHOD: MoM; \\Media of Maxima

  END\_DEFUZZIFY

## FCL Example (4): rule definition

RULEBLOCK first

AND:MIN;

ACCU:MAX;

RULE 0: IF (Our\_Health IS Near\_Death) AND (Enemy\_Health IS Near\_Death)  
THEN (Aggressiveness IS Fight\_Defensively);

RULE 1: IF (Our\_Health IS Near\_Death) AND (Enemy\_Health IS Good)  
THEN (Aggressiveness IS Run\_Away);

RULE 2: IF (Our\_Health IS Near\_Death) AND (Enemy\_Health IS Excellent)  
THEN (Aggressiveness IS Run\_Away);

RULE 3: IF (Our\_Health IS Good) AND (Enemy\_Health IS Near\_Death)  
THEN (Aggressiveness IS All\_Out\_Attack);

RULE 4: IF (Our\_Health IS Good) AND (Enemy\_Health IS Good)  
THEN (Aggressiveness IS Fight\_Defensively);

RULE 5: IF (Our\_Health IS Good) AND (Enemy\_Health IS Excellent)  
THEN (Aggressiveness IS Fight\_Defensively);

RULE 6: IF (Our\_Health IS Excellent) AND (Enemy\_Health IS Near\_Death)  
THEN (Aggressiveness IS All\_Out\_Attack);

RULE 7: IF (Our\_Health IS Excellent) AND (Enemy\_Health IS Good)  
THEN (Aggressiveness IS All\_Out\_Attack);

RULE 8: IF (Our\_Health IS Excellent) AND (Enemy\_Health IS Excellent)  
THEN (Aggressiveness IS Fight\_Defensively);

END\_RULEBLOCK

END\_FUNCTION\_BLOCK

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## HW for fuzzy systems

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PC and assembly, or C (C++, Java, ...) code

Standard microcontroller (compiled fuzzy code)

Fuzzy HW

- Fuzzy processor: almost only a RAM (everything is precomputed) with data acquisition (8 to 12 bit) and serial interface
- Standard microcontroller augmented with fuzzy instructions in the instruction set (e.g., fuzzyfy, defuzzyfy, ...)
- Fuzzy processor integrated with other devices (e.g., pwm generator, ...)