Transportation network user equilibrium assignment by ant colony systems with a variable trail decay coefficient

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Abstract: The paper deals with deterministic and stochastic user equilibrium (DUE and SUE respectively) two well known problems in the transportation field where the transportation demand has to be assigned to the network. In order to solve these problems a modified version of the ant colony system is proposed evolving ACS from discrete to continuous optimization. The ant colony meta-heuristic is adapted theoretically in order to take into account many aspects characterizing the transportation problem: multiple ODs (Origin-Destination), link congestion, non-separable cost link functions different user cost models including stochastic cost perception. Other improvements are introduced into the ACS algorithm to speed up convergence for selecting the minimum cost path and especially for getting a variable trail decay coefficient, ρ. The application of the propose algorithm to the Sioux Falls test network is finally reported.

Keywords: Transportation model, User equilibrium assignment, ant colony system.

1. INTRODUCTION

Except the very particular case of a non-congested network, route choice depends on congestion giving rise to a particular interaction between demand and supply. The best known approaches to analysis of the steady state are the deterministic user equilibrium (DUE) and the stochastic user equilibrium (SUE).

DUE was formulated by Wardrop (1972) as a criterion to find demand distribution on routes. This criterion states that journey time on all the routes actually used are equal or less than those which would be experienced by a single vehicle on any other route. This is based on strong assumptions that network travel times are deterministic for a given flow pattern and that all travelers are perfectly aware of the travel times on the network and always capable of identifying the shortest travel time route. To overcome the limits of the deterministic model, some researchers have proposed different SUE models (Daganzo and Sheffy, 1977) to relax the assumption of perfect knowledge of network travel times, allowing travelers to select routes based on their perceived travel times.

The ant colony system (ACS) is a particular implementation of the Ant Colony Optimization (ACO) meta-heuristic (Dorigo, M., Stützle, 2004; Dorigo and Blum, 2005). In (Dorigo and Blum, 2005) a comprehensive survey about ACO theory is presented reviewing all convergence results built up in the long period since ACO was introduced. Convergence is a crucial point tackled by many researchers and some important proofs can be found in (Gutjahr, 2000) for deterministic combinatorial optimization and in (Gutjahr, 2004) for the stochastic version with the expected value of a random variable as objective. In these papers problems are discrete and the existence of a solution is assumed always true both for the nature of the problem faced and for the search mechanism which implies, at infinitum, the possibility of exploring the whole set of solutions. To this extent, ACS has been successfully applied to many discrete optimization problems also in the transportation field. Typical applications of the ant colony system are Travelling Salesman Problem (Dorigo et al. 2004; Dorigo and Gambardella, 1997; Li and Gong, 2003), network routing (Di Caro, G., Dorigo, 1998), and the Quadratic Assignment Problem (Gambardella et al., 1999).

More recently, ant colonies have been applied to road traffic management (Bertelle et al., 2003) and resource allocation in transportation (Doerner et al., 2001). Also application to the network design problem have been faced successfully by the ant systems (Poorzahedy and Abulghasemi, 2005) which can tackle the difficulty of the combinatorial nature of that problem and the non convexity of its objective function.

A first attempt to extend ACO to continuous search domains is in (Socha, 2004; Socha and Blum, 2004). The underlying idea was developed and very recently presented by (Socha and Dorigo, 2008) showing how combinatorial ACO meta-heuristic can be adapted to continuous optimization without

any major conceptual change to its structure. The probabilistic choices of a solution made by ants define, instead of a discrete probability function, a probability density function (PDF) in the sense that an ant making its choice samples a PDF.

One of the first attempts to apply an ant system to the continuous domain in the transportation field is that of D'Acierno et al. (2006) who proposed an MSA (Method of Successive Averages) algorithm for a SUE simulation based on the ACO paradigm already presented in (Mussone et al., 2005). This work is particularly relevant and fundamental since it proves that ant systems are capable of solving a SUE formulation under a Logit hypothesis.

Other applications of ACO technique in the transportation field can be found in (Mussone et al., 2005; Matteucci and Mussone, 2006) where first attempts to apply ACO were proposed and analyzed with regards to their performance with respect to some internal parameters (e.g., the role of pheromone, of heuristics to accelerate convergence, and above all the stochastic nature of the technique were only superficially tackled). The point of view we proposed in those papers is strictly continuous (i.e. non combinatorial) and, for convergence proofs, can be theoretically based on the fixed point solution of a transportation network equilibrium.

The probabilistic nature of ant choice in performing the equilibrium solution defines a continuous set of probabilities which characterizes and differentiates this application to the other proposed at the present time. Another contribution is related to the capability of performing both DUE and SUE and, in this later case, by using whatever path choice model. In this sense this paper generalizes and extends the contribution by D'Acierno et al. (2006).

The aims of this paper are therefore manifolds and concern the application of ACS to cope with several aspects of DUE/SUE traffic assignment. We developed an algorithm, based on an ACS, able to search the equilibrium solution of the assignment problem especially when uniqueness of solution is not known. Ant Systems are proved to converge in probability to the optimal solution thus they can be used to explore the entire graph of paths while searching for a stochastic loading of flow. Experimentally we have validated this algorithm also on real network scenarios (though in this paper we report only the case of the Sioux Falls network) showing that it achieves good solutions, especially for the SUE assignment, with a reduced computing time with respect to traditional algorithms.

Moreover, the algorithm we propose requires no assumptions on the link cost functions or the user choice model so it is suitable, as we tested in the experiments, also for link cost functions with non-separable costs (that is the case of links ending with non signalized intersections such as T-intersections and roundabouts). These properties derive from the same mechanism used for finding solution, as explained later, which requires at each iteration a simple assignment (Deterministic Network Loading or Stochastic Network Loading) in a Monte Carlo approach when searching the best path. Non-separable cost functions at each iteration are treated as simple cost function, elasticity of demand implies only a new equation in the fixed point problem (other than costs or flow), multiclass assignment requires simply more ant colonies, one for each class and for each OD. All these features do not modify the kernel mechanism of the proposed version of ACO and make it suitable for a deeper analysis of congested networks.

2. THE TRAFFIC ASSIGNMENT PROBLEM

A transportation system may have different admissible states during time. The evolution mechanism is represented by the circular dependence between traffic demand, flows and costs (Fig. 1). In some cases the circular dependence evolves to steady conditions where demand flow and costs are mutually consistent. However it must be stressed that the proposed approach does not concern how the network reach the equilibrium condition (if any), but the equilibrium solution of an assignment model. In the following paragraphs some basic notations and formalization for the steady-state (equilibrium) case are introduced and discussed.

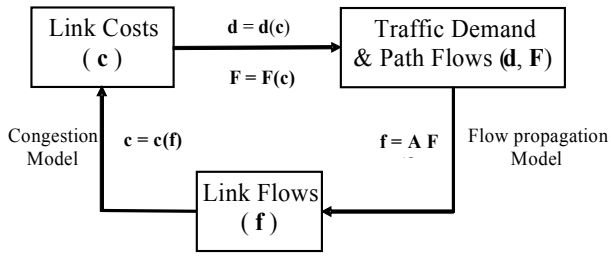

Fig. 1: Equilibrium relationship between traffic demand, flow and costs.

Given a directed graph G(V,E), with E being a set of N nodes and V a set of l arcs, we can identify a subset of ≤ N nodes in E and call them centroids. Let **d** be a vector with components representing the average number of trips going from centroid origin o to centroid destination d within a given time period. Each origin-destination flow generates on the network path flows F_i, with $i \in I_{od}$, where I_{od} is the subset of all admissible paths connecting the pair of centroids o and d. For a given link/edge $e \in E$, the sum of all path flows crossing this link is called the propagation model:

$$f_i = \sum a_{ik} F_k \qquad (1)$$

where a_{ik} is 1 if the link i is crossed by the path k and 0 otherwise. In matrix form

$$\boldsymbol{f} = \boldsymbol{A}\boldsymbol{F} \qquad (2)$$

where $\boldsymbol{F}$, and $\boldsymbol{f}$ are the vectors of path and link flows, respectively, and $\boldsymbol{A}$ is the link-path incidence matrix.

A model of a transportation system describes the behavior of traffic demand **d** and its relationship with link flows. By introducing a cost $c_e(\boldsymbol{f})$ for traveling on a certain link e, depending on the observed traffic $\boldsymbol{f}$, one can express traffic demand and the way it is distributed on links as a function of

the vector of costs $\boldsymbol{c}$, in particular the relationship between **F**, **f** and **d** becomes

$$\boldsymbol{F} = \boldsymbol{P}(\boldsymbol{c}(\boldsymbol{f}))\boldsymbol{d}(\boldsymbol{c}(\boldsymbol{f})) \text{ or } \boldsymbol{F}^* = \boldsymbol{P}(\boldsymbol{A}^T\boldsymbol{c}(\boldsymbol{A}^T\boldsymbol{F}^*))\boldsymbol{d}(\boldsymbol{A}^T\boldsymbol{c}(\boldsymbol{A}^T\boldsymbol{F}^*)) \quad (3)$$

$$\boldsymbol{f} = \boldsymbol{A}\boldsymbol{P}(\boldsymbol{c}(\boldsymbol{f}))\boldsymbol{d}(\boldsymbol{c}(\boldsymbol{f})) \text{ or } \boldsymbol{f}^* = \boldsymbol{A}\boldsymbol{P}(\boldsymbol{A}^T\boldsymbol{c}(\boldsymbol{f}^*))\boldsymbol{d}(\boldsymbol{A}^T\boldsymbol{c}(\boldsymbol{f}^*)) \quad (4)$$

where $\boldsymbol{P}$ is the path choice probability matrix, known also as path choice map, whose every element expresses the probability that traffic demand d_i (of the i-th od-couple) is routed on path k; $\boldsymbol{C}=\boldsymbol{A}^T\boldsymbol{c}$ represents the vector of path choices costs. Equations (2) and (3) or (4) describe the circular dependencies, the consistency of which is at the base of the equilibrium problem. As Fig. 1 shows, neither the way the system evolves nor how it reaches equilibrium is studied. It is assumed that when the system reaches equilibrium it becomes stationary, because demand is constant so its link performance only can modify the balance between demand and costs.

Equilibrium can be analyzed with a further condition on OD trip matrix elasticity. The OD trip matrix can be rigid, in the sense that the cost variation caused by congestion affects only the choice of path. This means that vector d is assumed invariant to link costs. Vector $\boldsymbol{f}^*$ or $\boldsymbol{F}^*$ are then defined by the equations:

$$\boldsymbol{F}^* = \boldsymbol{P}(\boldsymbol{C}(\boldsymbol{F}^*))\boldsymbol{d} \text{ where } \boldsymbol{C}(\boldsymbol{F}^*) = \boldsymbol{A}^T\boldsymbol{c}(\boldsymbol{A}\boldsymbol{F}^*) \quad (5)$$

$$\boldsymbol{F}^* = \boldsymbol{A}\boldsymbol{P}(\boldsymbol{C}(\boldsymbol{f}^*))\boldsymbol{d} \text{ where } \boldsymbol{C}(\boldsymbol{f}^*) = \boldsymbol{A}^T\boldsymbol{c}(\boldsymbol{f}^*) \quad (6)$$

Otherwise demand can be elastic; this means that it depends on congestion costs as well as on system attributes.

2.1 User equilibrium with rigid demand

The problem of finding vectors $\boldsymbol{f}^*$ and $\boldsymbol{F}^*$, under the assumption of a deterministic path choice model (DUE) in order to avoid some mathematical difficulties due to the fact that in the deterministic case (3) and (4) (even in the case of rigid demand) are multi-valued maps, is studied by means of formulations based on the variational inequalities:

$$\boldsymbol{C}(\boldsymbol{F}^*)^T(\boldsymbol{F}-\boldsymbol{F}^*) \geq 0 \ \forall \boldsymbol{F} \in S_F \quad (7)$$

where S_F is the set of admissible path flow vectors. Equivalent variational inequality models are based on link flow leading to

$$c(\boldsymbol{f}^*)^T(\boldsymbol{f}-\boldsymbol{f}^*) \geq 0 \ \forall \boldsymbol{f} \in S_f \quad (8)$$

where S_f is the set of admissible link flows.

The calculation of equilibrium link flow with rigid demand and symmetric Jacobian of cost functions is based on algorithms that solve the previous variational inequalities by an optimization model. One of the most famous is the Frank-Wolfe algorithm (FW) (Frank and Wolfe, 1956) from which many variants were derived.

When link costs are non-separable, the Jacobian may not be symmetric and other algorithms must be used, for example the diagonalization algorithm, which uses an approximation of the variational inequality by using separable cost functions. It is worth noting that this does not ensure the underlying equilibrium has a unique solution. DUE can be solved by the MSA approach as well (see for example Cascetta, 2001), presented in the next paragraph, though in many cases this can imply a reduction of performance from the point of view of convergence speed.

A stochastic equilibrium assignment is obtained by applying the equilibrium approach to the assignment of congested networks on the hypothesis of a probabilistic behavior of choice. SUE was first introduced by (Daganzo and Sheffy, 1977) and developed later in a limited form by Sheffi and Powell (1981); a first model for optimization was proposed by (Daganzo, 1983) for symmetric SUE. In (Daganzo and Sheffy, 1977) the authors applied the method of fixed point in order to solve deterministic or stochastic equilibrium. A plain complete review of SUE models is presented by (Cascetta, 2001). In (Cantarella, 1997) a general formulation of the fixed point for multi-mode multi-user equilibrium with elastic demand is presented both for SUE and DUE. Conditions for existence and uniqueness of solutions are stated and recalled in the next sections.

Other important results about solution of a SUE problem can be found in (Bar-Gera and Boyce, 2006) where authors studied a travel forecasting model by applying the MSA with a constant step size and obtaining convergence performance than the classical MSA with a decreasing 1/k step size. This feature is particularly relevant for the purpose of this work since updating rule of ants generally have a constant step size. Another recent application of a variant of MSA (a weighted version) can be found in (Liu et al., 2007).

3. ANT SYSTEM

To better understand the functioning of our algorithm for traffic assignment, we divide the exposition into two parts: the use of an ACS to solve the shortest path problem and then we introduce the congestion phenomena.

The first problem is similar to the Travelling Salesman Problem (TSP) (Dorigo and Gambardella, 1997a; Dorigo and Gambardella, 1997b; Dorigo et al., 2002; Li and Gong, 2003), solved by using the classical ACO meta-heuristic: the Ant System (AS).

In TSP, the ant-decision table defined for node i as $A_i = [a_{ij}(t)]|N_i|$ of node i is usually obtained by the composition of the local pheromone trail values with local heuristic values as follows:

$$a_{ij}(t) = \frac{[\tau_{ij}(t)]^{\alpha}[\eta_{ij}]^{\beta}}{\sum_{l \in N_i}\{[\tau_{il}(t)]^{\alpha}[\eta_{il}]^{\beta}\}}, \forall j \in N_i \quad (9)$$

where $\tau_{ij}(t)$ is the amount of pheromone trail on link (i, j) at time t, $\eta_{ij} = 1/d_{ij}$ is the heuristic value used in the TSP problem for the movement from node i to node j, Ni is the set of neighbors of node i, and α and β are parameters controlling the relative weight of pheromone trail and heuristic value, respectively. The probability an ant k chooses to go from city i to city $j \in N_k$ when constructing its journey, solution of the TSP, at the t-th algorithm iteration, is:

$$p_{ij}^{k}(t)=\frac{a_{ij}(t)}{\sum_{l\in N_i^k} a_{il}(t)} \tag{10}$$

where $N_i^k \subseteq N_k$ is the set of nodes connected to node *i* that ant k has not visited yet (nodes in N_i^k are selected from those in N_i by using ant memory M^k). After all the ants have completed their tour, pheromone evaporation on all links is triggered, and, after that, each ant k deposits a quantity of pheromone $\Delta\tau_{ij}^{k}(t)$ on each used link:

$$\Delta\tau_{ij}^{k}(t)=\begin{cases}1/L^k(t) & if \quad arc(i,j)\in T^k(t)\\ 0 & if \quad arc(i,j)\notin T^k(t)\end{cases} \tag{11}$$

where $T^k(t)$ is the tour made by ant *k* at iteration *t*, and $L^k(t)$ is the tour length. It is clear from (11) that the value $\Delta\tau_{ij}^{k}(t)$ depends on ant performance: the shorter the tour made, the greater the amount of pheromone deposited.

In practice, the addition of new pheromone by ants and pheromone evaporation are implemented by the following rule applied to all the links:

$$\tau_{ij}(t)\leftarrow(1-\rho)\tau_{ij}(t)+\rho\,\Delta\tau_{ij}(t) \tag{12}$$

where $\Delta\tau_{ij}(t)=\sum_{k=1}^{m}\Delta\tau_{ij}^{k}(t)$, *m* is the number of ants at each iteration *k* (maintained constant), and $\rho\in(0,1]$ is the pheromone trail decay coefficient. The initial amount of pheromone $\tau_{ij}(0)$ is set to the same small positive constant value τ_0 on all links in order to randomly explore the network at the first iteration of the algorithm.

ρ can be constant or decreasing according to a certain law in order to accelerate (and to guarantee) convergence. In this paper the following general law is adopted:

$$\rho=\rho_0 e^{-sk}$$

where ρ_0 (≤ 1) is the value at k=0 or when the descent coefficient *s* is nil. The higher s the steeper the decay of ρ.

3.1 Ant Colony System for Traffic Assignment (ACS-TA)

To tackle the congestion in traffic the Ant System algorithm, in the Ant Colony System (ACS) (Dorigo and Gambardella, 1997a) just described has been modified to mimic user behavior in a transportation network. An ant decides to travel on a path using the information left before by other ants, and then it distributes a new quantity of pheromone in function of the "goodness" of the path. In order to effectively explore the space of solutions there must be a relationship between pheromone and flow, because ants deposit pheromone not flow.

For each link, a quantity of flow proportional to the pheromone present on it could be associated. Then, after pheromone distribution, the new proportional flow assignment implies a variation of costs that leads the ants following to have a different evaluation of paths. This implies a positive or negative feedback that takes the system to an equilibrium state, for which a variation in flow assignment implies a variation in pheromone distribution that re-establishes the equilibrium. All this can be viewed in the same manner as the circular relationships of equilibrium between traffic demand, flows and costs explained previously where pheromone now substitutes traffic demand (see Fig. 2).

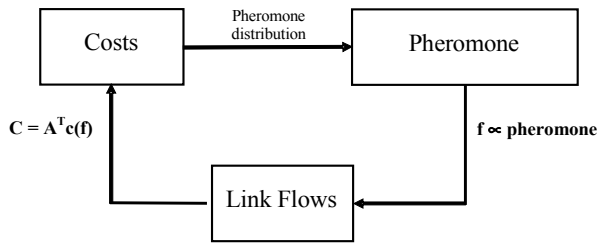

Fig. 2: Relationships between pheromone distribution, flows and costs.

This view is commonly proposed for the SUE case but it can be extended to the DUE case as well being it a particular case of SUE though it is generally solved by a FW approach.

Similarly to what is done in (11), at every iteration step each ant deposits a quantity of pheromone $\Delta\tau_{ij}^{k}(t)$ on each link equal to:

$$\Delta\tau_{ij}^{k}(t)=\begin{cases}1/C^k(t) & if \quad arc(i,j)\in T^k(t)\\ 0 & if \quad arc(i,j)\notin T^k(t)\end{cases} \tag{13}$$

where $T^k(t)$ is the tour made by ant *k* at iteration *t*, and $C^k(t)$ is its cost. For every od-couple there is an ant colony, with its own nest (centroid O) and a food source (centroid D). Every ant of the same colony distributes pheromone of the same type, so that the ants in that colony can recognize and follow only paths that lead to the same food source.

Every ant colony is independent and its ants have to route a quantity of flow equal to the corresponding flow demand from an origin O to a destination D. This leads to have on link (i, j) at time t a quantity of flow equal to:

$$f_{ij}(t)=\sum_{c=1}^{N_{OD}} d_c\,\frac{\tau_{ij}^{c}(t)}{\tau_{FS}^{c}(t)} \tag{14}$$

where d_c is the flow demand of colony *c*, N_{OD} is the number of colonies, $\tau_{ij}^{c}(t)$ is the quantity of pheromone of colony *c* on link(i,j) and $\tau_{FS}^{c}(t)$ is the sum of pheromone quantities on the links of the forwarding star of node *i* of colony *c*. It is plain that other different forms of equation 14 can be formulated by taking a different function for pheromone: for example the best known Logit model can be obtained by using the exponential function of pheromone. In any case all these variations lead to a stochastic model but it can be extended to the DUE case as well being it a particular case of SUE though

it is generally solved by a FW approach.

To improve the estimate of the path cost made by the ants and assure the more of them traverse useful path we have introduced a couple of heuristics in order to speed up algorithm convergence. We recall here, that an ant decides to travel on a path using the information left before by other ants. In particular, an ant must follow paths leading to the food source (i.e., its destination) belonging to its colony, and so it has to pay attention only to the information (pheromone) left by the other ants of the colony. According to this vision we have a colony for each origin destination pair.

```
while convergence on pheromone not reached do
    forall colony do
        forall ant do
            Find the path from origin to destination (sequence of nodes and links
            Deposit pheromone on the path;
        end
    end
    Evaporate the pheromone;
    Assign flow according to pheromone;
    Calcolate costs on links;
end
Output flow;
```

Fig. 3: The pseudocode of the Ant Colony System for Traffic Assignment (ACS-TA) in the SUE version.

Let focus on a single colony; the ant-decision table $A_i^c = [a_{ij}^c(t)]_{|N_i|}$ of node i and colony c can be obtained by the composition of the local pheromone trail values with an heuristic weight of the minimum cost path as follows:

$$a_{ij}^c(t) = \frac{[\tau_{ij}^c(t)]^\alpha [w_{ij}(t)]^\beta}{\sum_{l \in N_i} [\tau_{il}^c(t)]^\alpha [w_{il}(t)]^\beta}, \forall j \in N_i \tag{22}$$

where $\Delta\tau_{ij}^c(t)$ is the amount of pheromone trail of colony c on link (i,j) at time t and $w^c_{ij}(t)$ is a weight value of link (i,j), set as follows:

$$w^c_{ij}(t) = \begin{cases} w & \text{if} \quad arc(i,j) \in \quad shortest\_path \\ 0 & \quad otherwise \end{cases} \tag{23}$$

As described in the previous section, the pheromone trail for the specific colony is updated by $\Delta\tau_{ij}^c(t)$ according to the inverse cost of the path found. The use of w induces a heuristic bias to the ACS-TA algorithm to speed up the convergence process through the calculation of the minimum path for each colony. To take into account the cost circular relationship between cost and flow, w needs to be iteratively updated with the value of $1/C(t)$ of the shortest path requiring, to find the optimal values of $w^c_{ij}(t)$, the computation of the minimum path for every *od*-couple at each iteration.

As a side effect, this heuristic assures that an ant can decide to choose links that belong to the minimum path, even if there is no more pheromone on them due to the evaporation procedure. This can be seen as a probability biasing which always gives an ant a chance of finding a good solution.

Taking into account the deterministic or stochastic nature of user decision we introduced a further improvement in the ant decision table. The value of $\tau_{ij}^c(t)$ used in (15) may be the exact amount of pheromone released by ants or the perceived one extracted according to a certain distribution. This turns out to bias the ant search strategy toward a deterministic user model in the case the exact pheromone value is used (i.e., the user choice is the one that deterministically select the minimum cost), as well as a stochastic one when the pheromone trail is treated as a random variable.

Again, this is a heuristic to speed up the algorithm when dealing with SUE providing a better exploration of alternative solutions and thus a better estimate of the perceived cost and its distribution In doing the sampling we can use a normal distribution to represent a symmetric uncertainty on the actual estimate of link cost, or a lognormal distribution modeling to represent an asymmetric optimistic uncertainty.

Again regarding the ant choice model, it is correct that (1) and (2) analyze the total flow present on a link due to all colonies, but this does not affect the choice mechanism of ants which is absolutely local. This might seem in contradiction when extending the approach to non separable cost functions, but in this case it must be considered that updating of flow and then of costs occurs at each iteration (during a deterministic or stochastic network loading, see Cascetta, 2001) with certain fixed values of flow.

A final remark is about the stopping condition. Convergence criteria for ACS are discussed in (Dorigo and Blum, 2005) and they are usually based on the following test:

$$\max_i \left| \frac{\left(y_i^t - y_i^{t-1}\right)}{y_i^{t-1}} \right| < \varepsilon \tag{24}$$

where y_i^{t-1} is the value of variables on link i at iteration (t-1), y_i^t is the value of the same variable on link i at iteration t and e is a threshold value. The variable considered for this work is the link cost, the link flow or the total cost as well; we also tested the algorithm convergence on pheromone since it represents the averaged variable of the fixed point problem and it is strictly related to link flow as explained in the previous section.

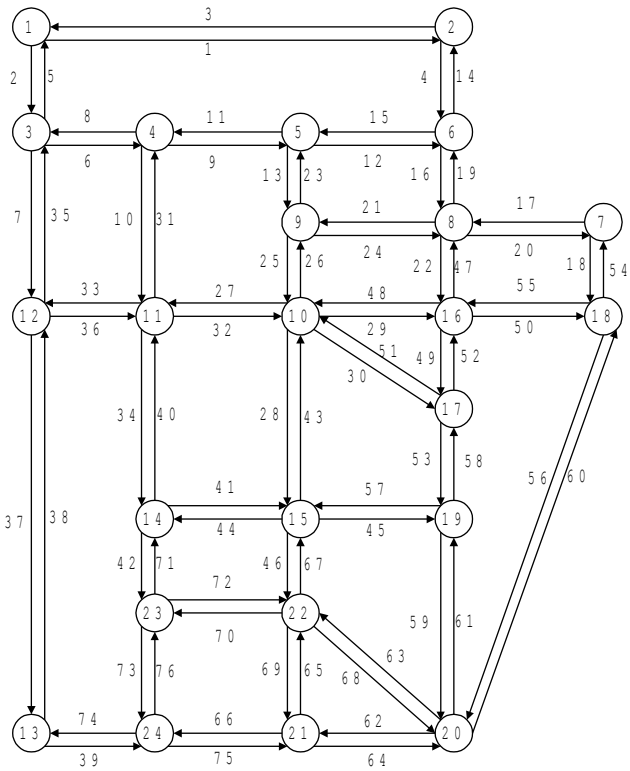

Fig. 4: The Sioux Falls test network.

4. RESULTS

Experiments were carried out on the Sioux Falls test network (Fig. 4) in the version proposed by LeBlanc (LeBlanc et al., 1975). This network has 24 centroids and 76 links. Demand is predetermined; optimal objective function (that is the total cost at equilibrium) is 42.31 and the solution of flow and cost on links is also known. Since the best solution is known and therefore comparisons can be done with different approaches it makes sense to use this network though is not considered a good representation of the real city.

In this work we are interested in understanding how the trail decay value ρ and its decaying rule affect the performances of the ACS-TA algorithm in terms of convergence. Nine different combinations of ρ_0 and s with 1000 iterations are investigated leading to the following intervals of variation for ρ: [0.1 0.1; 0.1 0.01; 0.1 0.001; 0.5 0.1; 0.5 0.01; 0.5 0.001; 0.9 0.1; 0.9 0.01; 0.9 0.001].

The higher the first value or the lower the second one the steeper the decay of ρ. Five different seeds are also used for random extractions and results are averaged on those obtained values. Comparison are carried out with the final solution obtained by applying the Frank Wolfe (FW) algorithm and the stochastic MSA (Method of successive Averages) as implemented in the Ciudadsim (http://www-rocq.inria.fr/metalau/ciudadsim/) software suite.

Solutions are obtained with two different forms for demand distribution in Equation 14: the first is just the ratio of pheromone values like in the equation 14, the second uses the exponential transformation of pheromone values leading to a Logit model, as proven by D'Acierno et al. (2006).

In Fig. 5, Fig. 6 and Fig. 7 results for a constant ρ are reported. Fig. 5 shows total cost for each iteration and speed of convergence. Fig. 6 and Fig. 7 show the values of cost and flow for each link according to the last iteration.

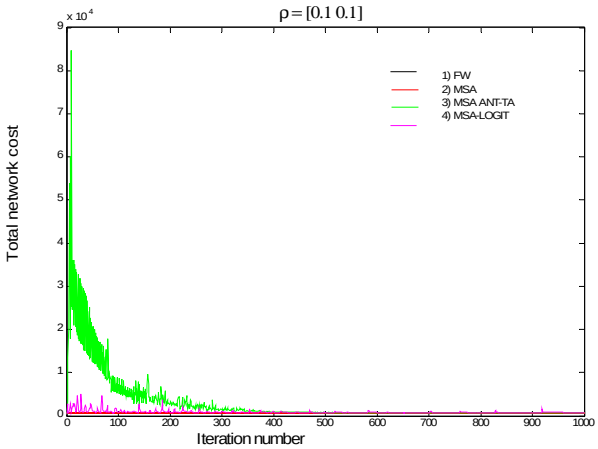

Fig. 5: Total cost for ρ: [0.1- 0.1] = costant.

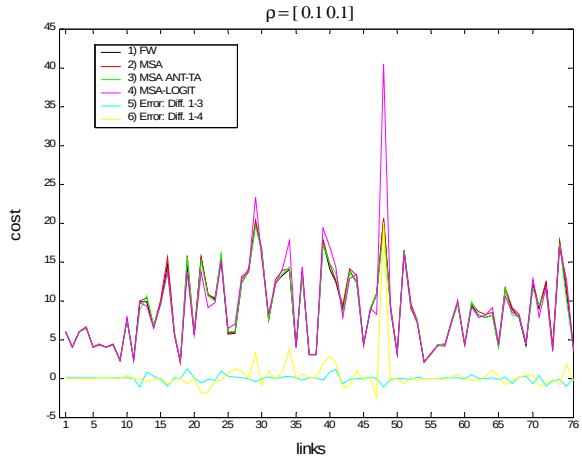

Fig. 6: Cost for each link for ρ: [0.1- 0.1] = costant.

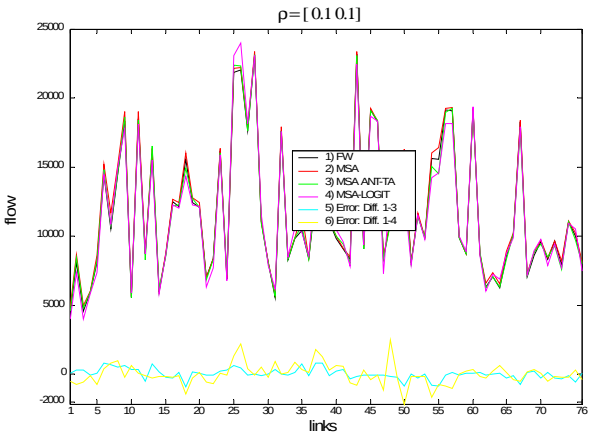

Fig. 7: Flow for link ρ: [0.1- 0.1] = costant.

In Fig. 8, Fig. 9 and Fig. 10 results for a ρ varying from 0.1 to 0.001 are reported. Fig. 8 shows total cost for each iteration and the speeder convergence with respect to the previous case. Fig. 9 and Fig. 10 show the values of cost and flow for each link according to the last iteration.

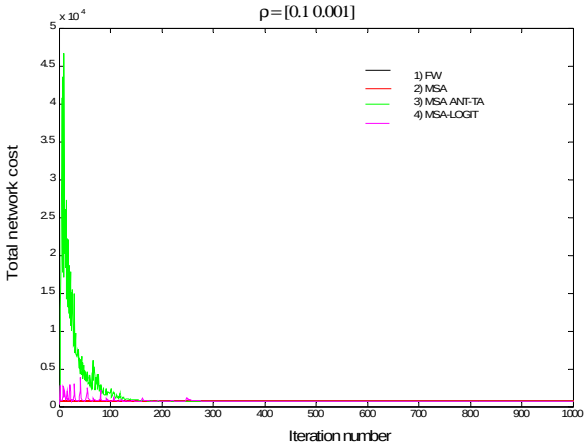

Fig. 8: Total cost for ρ: [0.1- 0.001].

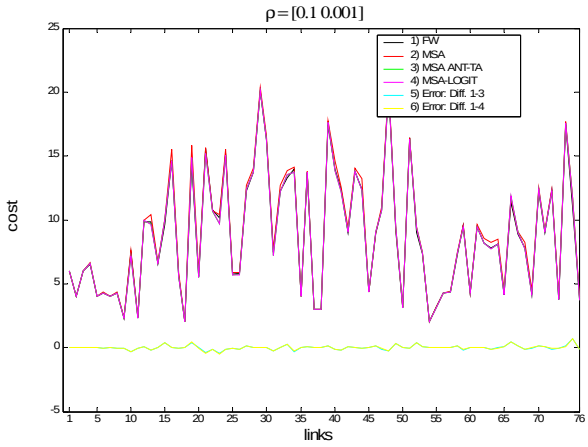

Fig. 9: Cost for each link for ρ: [0.1- 0.001].

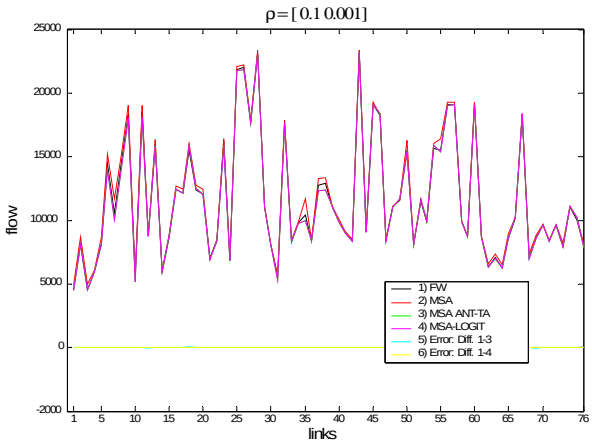

Fig. 10: Flow for link ρ: [0.1- 0.001].

Percentage error calculated for the last iteration is lower than 0.1% both for costs and for flow on links. Other experiments with higher values of ρ_0 show a slower convergence.

5. CONCLUSIONS

ACS is proposed in order to solve the equilibrium assignment problem in transportation networks. The modified version of this heuristic is suitable for application in almost all real cases due to its versatility and without assuming simplifying hypotheses. The solution found by ACS does not depend on the shape of the objective function and so the particular cases of non-separable cost link functions or multi-class demand can also be tackled easily and successfully.

Applications show a computation time that is also short enough (though not comparable to the speed of the Frank-Wolfe algorithm in the case of DUE) in complex networks and it can be improved through a parallel programming that is easy enough to apply thanks to the similar nature of ACS which is intrinsically parallel.

The impact of pheromone decay and variable trail decay coefficient is analyzed and we can suggest that its effects can depend strongly on network structure and cost functions. Generally we expect that by increasing the initial value of ρ, oscillations increase but this holds true only if the feasible set of paths is wide. Usually a few iterations are sufficient for the algorithm to converge. On equal number of iterations starting from lower values of ρ or increasing the descent coefficient improves performance: oscillations are reduced and convergence is obtained earlier. In general a variable decay coefficient ρ increases performance by increasing convergence speed while it does not affect the quality of the result.

More research is necessary to investigate convergence mechanisms especially when the existence and uniqueness of convergence cannot be theoretically demonstrated. Moreover, other models of cost perception, for example with a variance distribution function of its mean, could be tested together with

elastic demand and multi-class assignment.

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