

ROBOTICS (27/06/2016)

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The exam will be graded iff the following recommendations have been taken into account:

- Write clearly so that the teacher can easily understand your answers
- Write your name, surname, and student id on each sheet you deliver for evaluation
- For each exercise/question report clearly the number and sub-number (if present)
- You are not allowed to use any programmable device (e.g., smartphone, calculator, etc.)
- You can use pen or pencil, paper will be provided, you cannot use notes or books

Exercise 1 (Algorithm)

Let's consider how trajectory planning works, in particular:

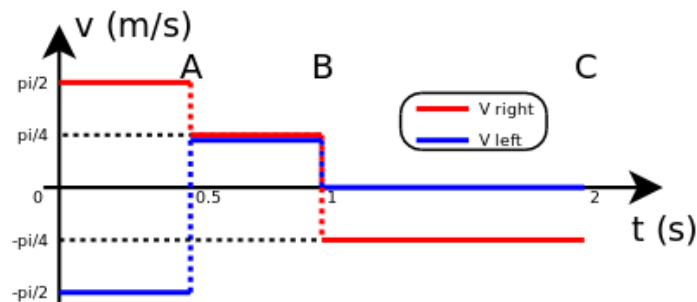
- a) Describe what is trajectory planning all about
- b) Describe how graph-based search planning works
- c) Describe the A* algorithm for graph-based search
- d) How can motion primitives improve on basic graph-based planning?

For the solution of this exercise, please refer to course slides.

Exercise 2 (Numerical Exercise)

Consider a differential robot base with a 50cm wheel base:

- a) Provide the direct kinematics formulas for a differential drive robot
- b) Assuming the robot starts in (0,0,0) compute the robot pose at times A, B and C according to the following actuations (numbers are strange, but computation should be easy)



For the solution of point a), please refer to course slides (in particular to the formulas describing linear and angular speed as a function of left and right wheel speed, the location of the ICC, the radius of rotation, etc.

For the solution of point b)

A) The first movement is an in place rotation with

$$\omega = (V_r - V_L)/L = \left(\frac{\pi}{2} + \frac{\pi}{2}\right)/0.5 = \pi * 0.5 = 2\pi$$

the duration of the in place rotation is 0.5s thus the final θ is equal to

$$\theta_A = \theta_0 + \omega * \Delta t = 0 + 2\pi * 0.5 = \pi$$

and the final position is $[0 \ 0 \ \pi]$

B) The second movement is a straight movement with

$$v = \frac{(V_r + V_L)}{2} = \frac{\frac{\pi}{4} + \frac{\pi}{4}}{2} = \frac{\pi}{4}$$

the robot starts from $[0 \ 0 \ \pi]$ facing left and moves for 0.5s along the x axis thus moving to

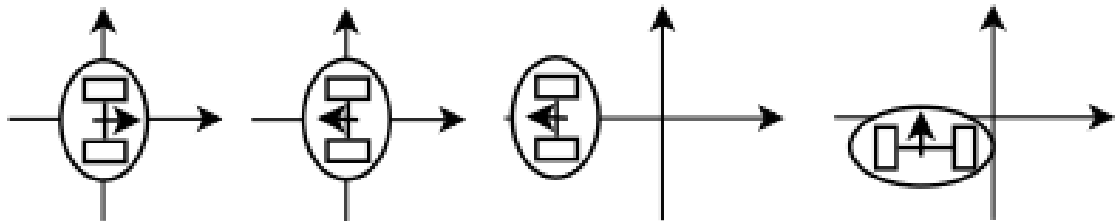
$$x_B = x_A + v * \Delta t = 0 - \frac{\pi}{4} * 0.5 = -\pi/8$$

the final position is thus $[-\pi/8 \ 0 \ \pi]$

C) The third movement can be computed analytically or derived through reasoning. The reasoning comes from the fact that the robot rotates around the left wheel (which is still) and perform an arc of circumference at the angular speed of

$$\omega = (V_r - V_L)/L = (-\frac{\pi}{4} + 0)/0.5 = -\pi/4 * 0.5 = -\pi/2$$

This act of circular movement is performed from $[-\pi/8 \ 0 \ \pi]$ and it lasts for 1s ending in $[? \ ? \ \pi/2]$, i.e., the robot is now pointing upward. The left wheel has not moved so it is still where it was originally, i.e., $[-\frac{\pi}{8} - 0.25]$, and thus also the barycenter of the robot has the same y position. From this we get $[? \ -0.25 \ \pi/2]$. To compute the x position of the baricenter, consider the fact that the robot has moved a quarter of circular trajectory backward so the x has been decreased by the radius of this movement, i.e., 0.25m (check the picture). Thus the final pose is $[0.25 - \frac{\pi}{8} - 0.25 \ \pi]$.



In case we want to apply the analytical formulas let recall

$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} \cos(\omega \cdot \Delta t) & -\sin(\omega \cdot \Delta t) & 0 \\ \sin(\omega \cdot \Delta t) & \cos(\omega \cdot \Delta t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x - CIR_x \\ y - CIR_y \\ \theta \end{bmatrix} + \begin{bmatrix} CIR_x \\ CIR_y \\ \omega \cdot \Delta t \end{bmatrix}$$

We know the CIR is on the left wheel, i.e., is $[-\frac{\pi}{8} - 0.25]$ and the rotation angular speed is

$$\omega = (V_r - V_L)/L = (-\frac{\pi}{4} + 0)/0.5 = -\pi/4 * 0.5 = -\pi/2$$

we obtain

$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} \cos(-\pi/2) & -\sin(-\pi/2) & 0 \\ \sin(-\pi/2) & \cos(-\pi/2) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -\frac{\pi}{8} - (-\frac{\pi}{8}) \\ 0 - (-0.25) \\ \pi \end{bmatrix} + \begin{bmatrix} -\frac{\pi}{8} \\ -0.25 \\ -\frac{\pi}{2} \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ \theta' \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0.25 \\ \pi \end{bmatrix} + \begin{bmatrix} -\frac{\pi}{8} \\ -0.25 \\ -\frac{\pi}{2} \end{bmatrix} = \begin{bmatrix} 0.25 - \frac{\pi}{8} \\ -0.25 \\ \frac{\pi}{2} \end{bmatrix}$$

Exercise 3 (Theory)

Let's consider the Simultaneous Localization and Mapping problem; answer the following questions

- How Simultaneous Localization and Mapping (SLAM) differs from Localization?
- How Simultaneous Localization and Mapping (SLAM) differs from Mapping?
- Describe what is the difference between online and offline SLAM
- Provide the derivation of the recursive Bayes Filter for online SLAM

For the solution of this exercise, please refer to course slides.

Exercise 4 (ROS)

Describe in details the roles of the ROS Master in a ROS architecture (what it is, what it does).

For the solution of this exercise, please refer to course slides.

Exercise 5 (Other)

What is the C-Space? Why and how it is used? Provide also a simple example to clarify your answer.

For the solution of this exercise, please refer to course slides.