Autonomous navigation of vehicles with single-track models using MPC

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Can we apply the approach used for unycicle robots also to more complicated vehicles?

E.g., a Yamaha Grizzly YMT 700 ATV (All Terrain Vehicle)

Outline

- Model and feedback linearization
- Goals and constraints
- Some simulation results
- Robustness issues

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Model and feedback linearization

$$
\begin{cases}
\dot{\psi} = r \\
\dot{r} = \frac{bCr - aC_f}{I_z z}\beta - \frac{a^2 C_f + b^2 C_r}{v I_z z} r + \frac{a C_f}{I_z z}\delta \\
\dot{\beta} = -\frac{C_f + C_r}{vm}\beta + \frac{C_r b - C_f a - mv^2}{mv^2} r + \frac{C_f}{mv}\delta \\
\dot{v} = a \\
\dot{x} = v\cos(\beta + \psi) \\
\dot{y} = v\sin(\beta + \psi)
\end{cases}
$$

Yaw angle

Yaw angle with respect to the center of mass

Steering angle (input)

Cornering stiffness coefficients

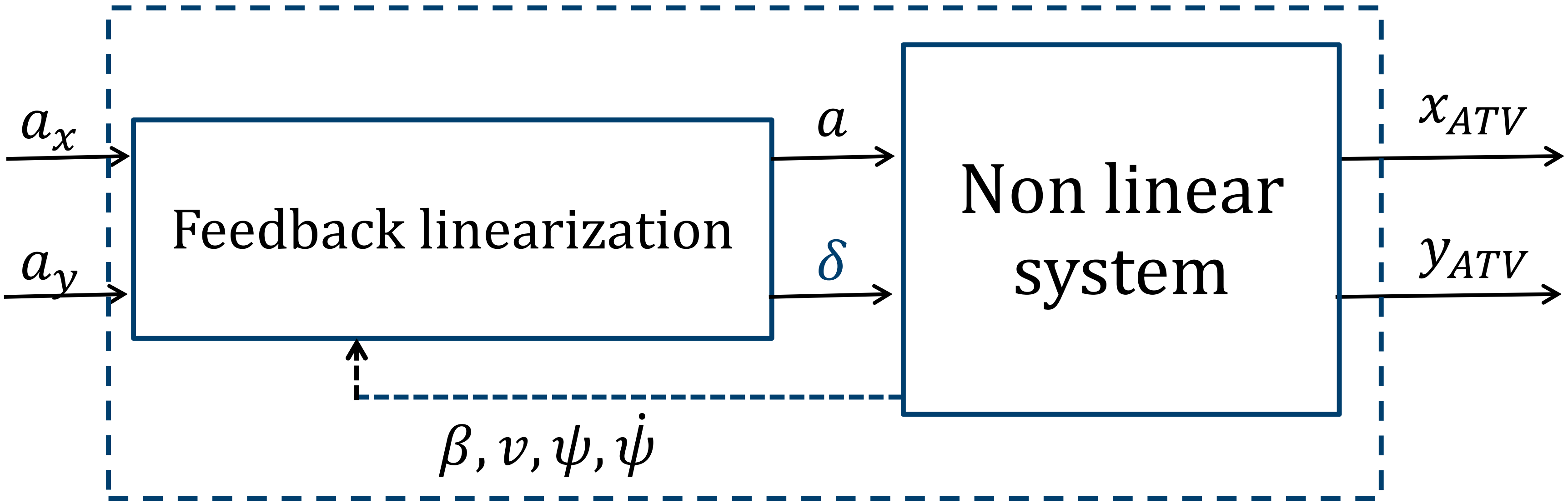
Distances of the center of mass with respect to front and rear axes

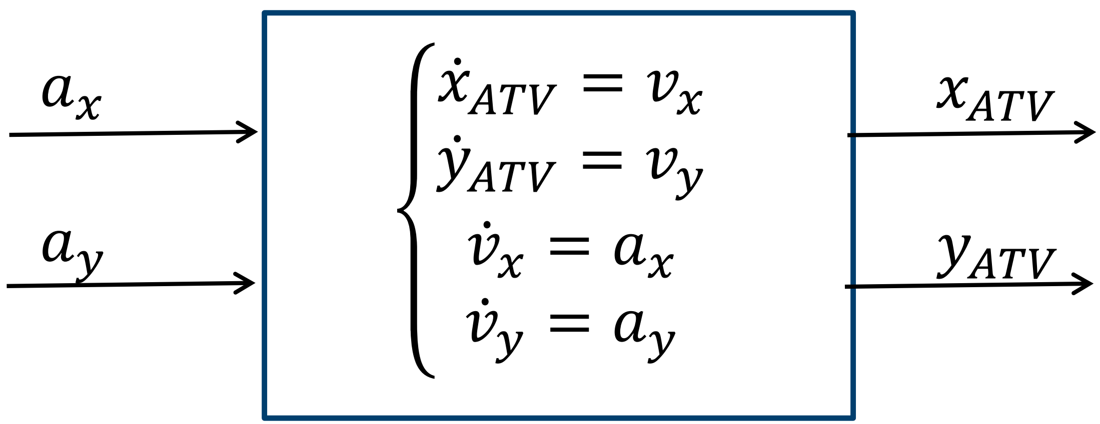
Longitudinal acceleration (input)

→ Can we still resort to ***feedback linearization?***

Model and feedback linearization – first approach

We can still obtain a feedback-linearized model of the type





Model and feedback linearization

Feedback-linearized model:

$$\begin{cases} \dot{x}_{ATV} = v_x \\ \dot{y}_{ATV} = v_y \\ \dot{v}_x = a_x \\ \dot{v}_y = a_y \end{cases}$$

Discrete time feedback-linearized model (sampling time τ)

$$\begin{bmatrix} x_{ATV}(k+1) \\ v_x(k+1) \\ y_{ATV}(k+1) \\ v_y(k+1) \end{bmatrix} = \begin{bmatrix} 1 & \tau & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \tau \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_{ATV}(k) \\ v_x(k) \\ y_{ATV}(k) \\ v_y(k) \end{bmatrix} + \begin{bmatrix} \tau^2/2 & 0 \\ \tau & 0 \\ 0 & \tau^2/2 \\ 0 & \tau \end{bmatrix} \begin{bmatrix} a_x(k) \\ a_y(k) \end{bmatrix}$$

$$x(t+1) = Ax(t) + Bu(t)$$

Output: position

$$\begin{bmatrix} x_{ATV}(k) \\ y_{ATV}(k) \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_{ATV}(k) \\ v_x(k) \\ y_{ATV}(k) \\ v_y(k) \end{bmatrix} = Cx(k)$$

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Goals and constraints

To fulfill this, we must include, in the MPC problem, the following ingredients:

- **Constraints**:
  - Collision avoidance constraints with respect to external walls
  - Collision avoidance constraints with respect to (fixed) obstacles
  - Constraints for fulfilling operational limitations (e.g., maximum speed, etc)
- **Cost function**:
  - For approaching the goal position
  - To prevent deadlock solutions with fixed obstacles
- Proper **terminal cost function and terminal constraints** for guaranteeing recursive feasibility.

Goals and constraints

All these ingredients are basically the same as in the case of the unycicle model.

In this case, however, there are more operational constraints:

- Accelleration
- Speed
- Steering angle
- Rate of variation of the steering angle

The latter are, e.g.,

$$-\delta_{max} \leq \delta(k) \leq \delta_{max}, \qquad k = t, \ldots, t+N-1$$

$$-\Delta\delta_{max} \leq \Delta\delta(k) \leq \Delta\delta_{max}, \qquad k = t, \ldots, t+N-1$$

⇨ They can be included as linear constraints using linearization techniques

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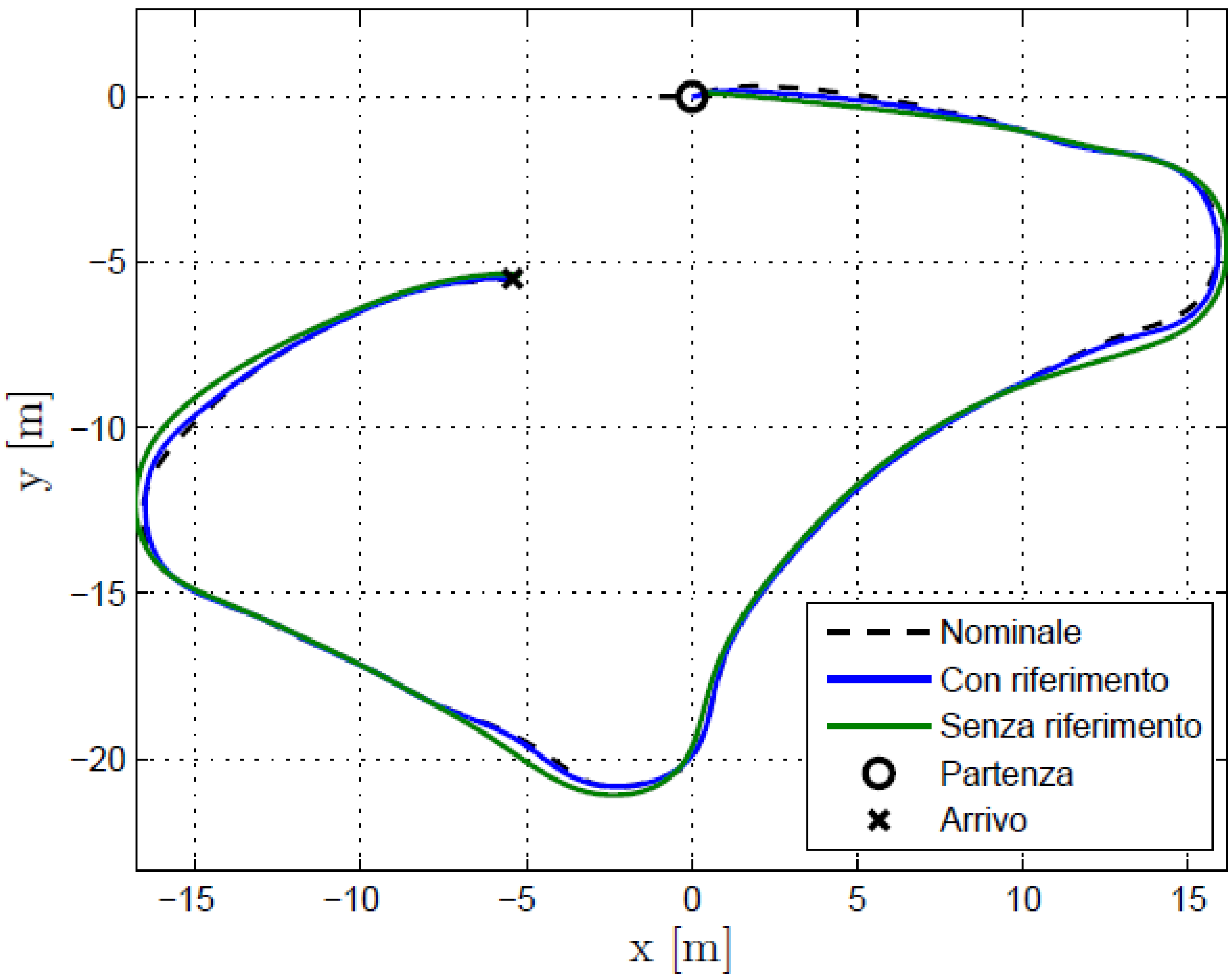
Some simulation results (Simulink model)



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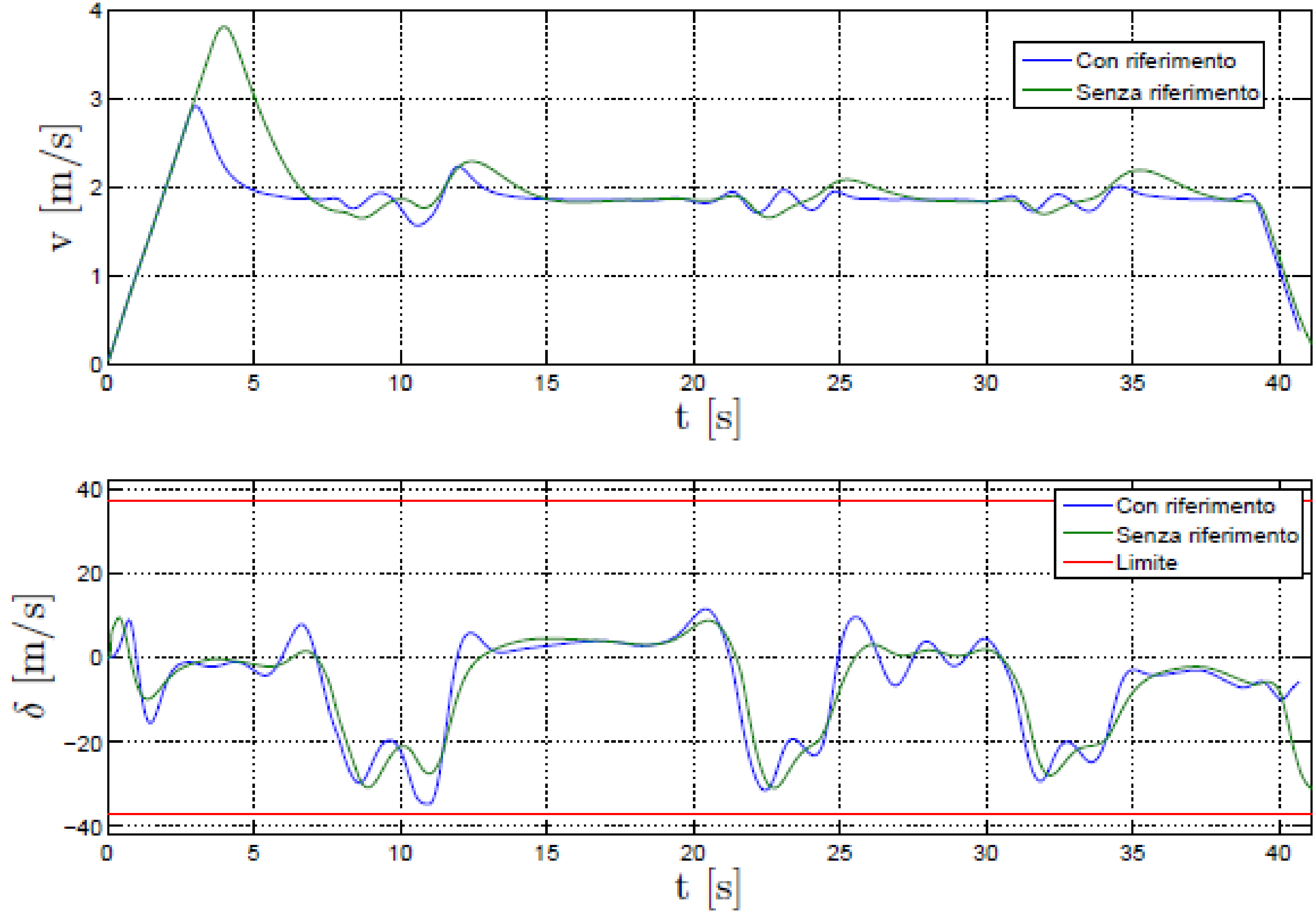

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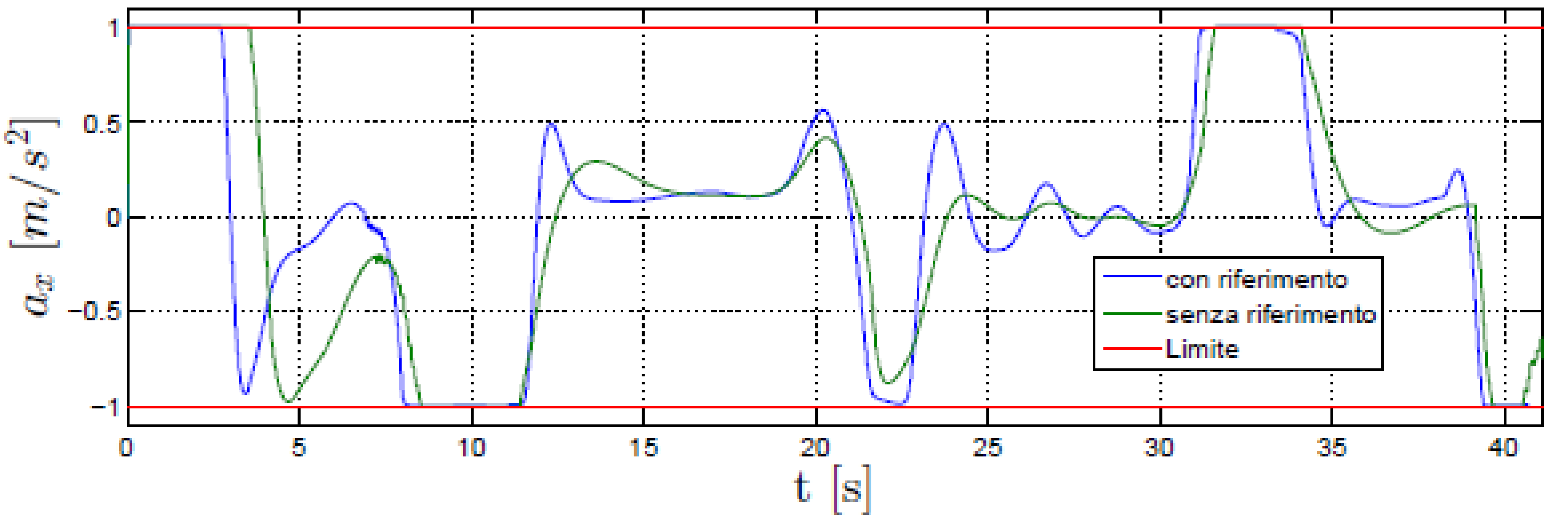

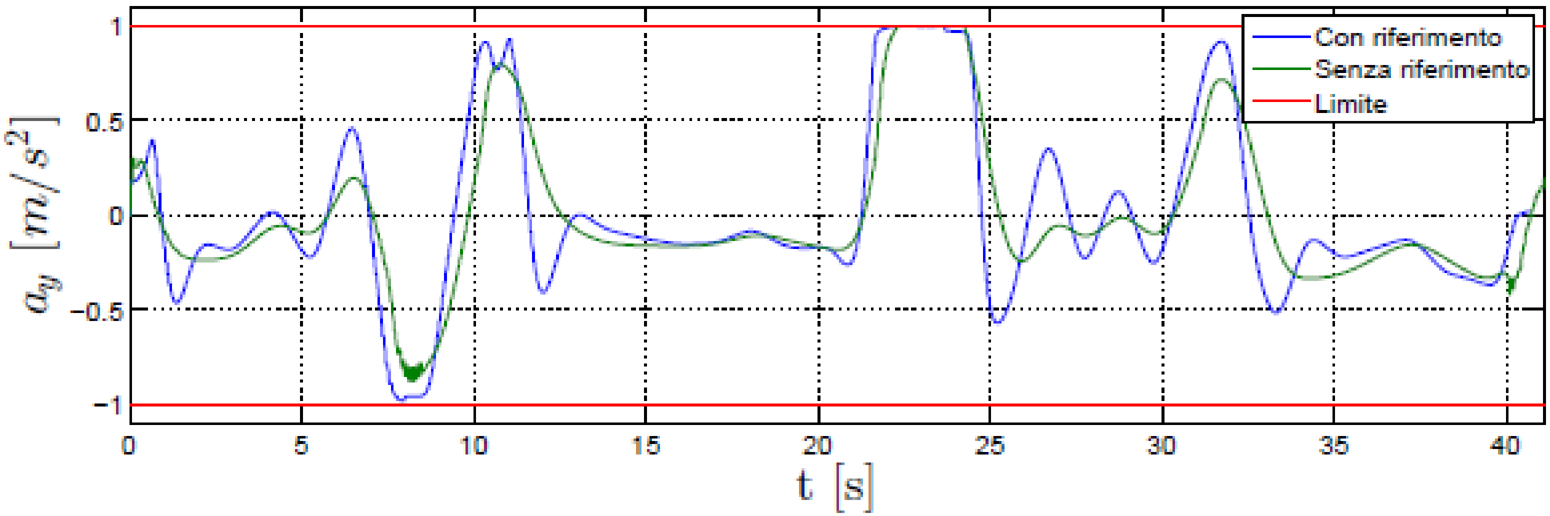

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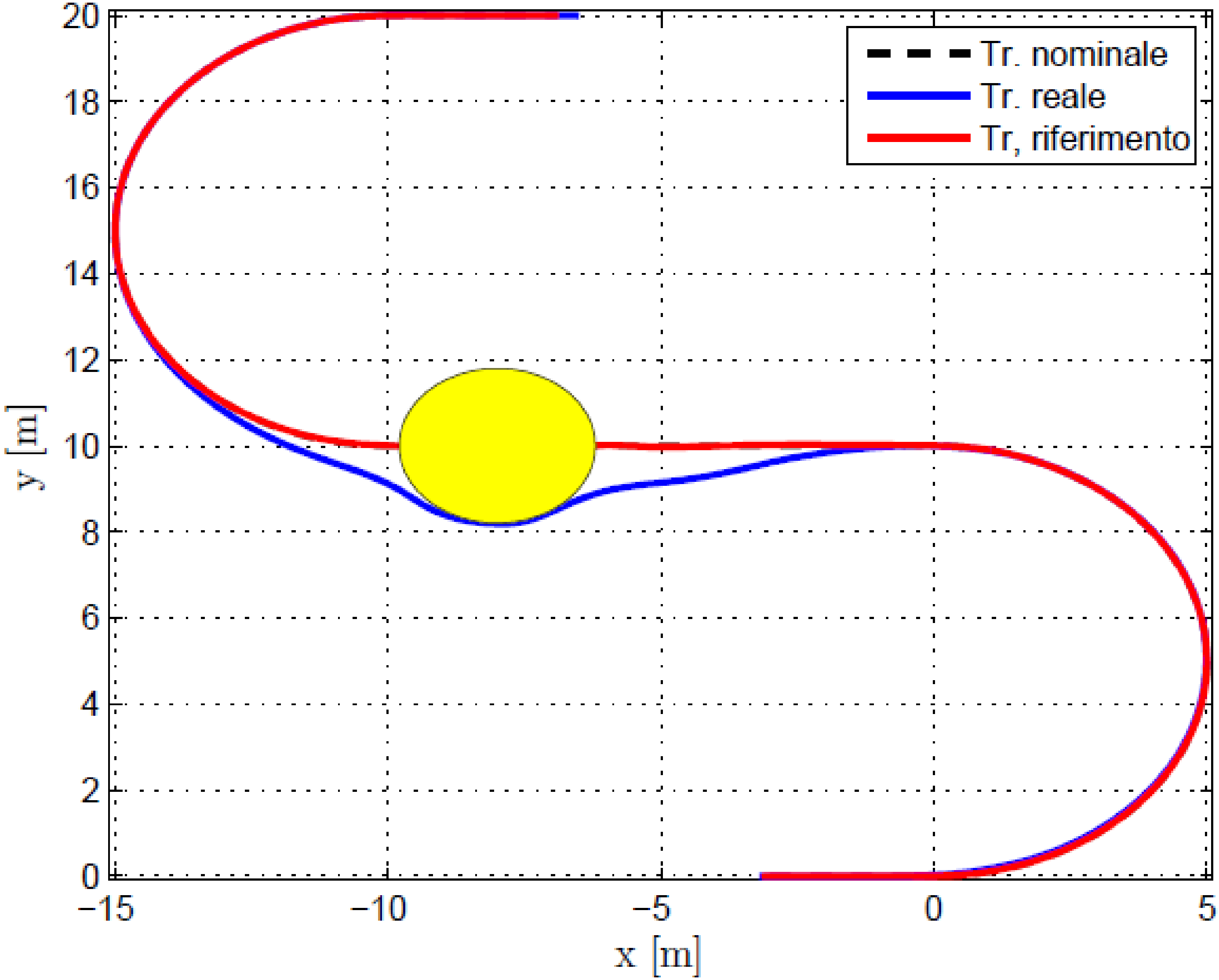

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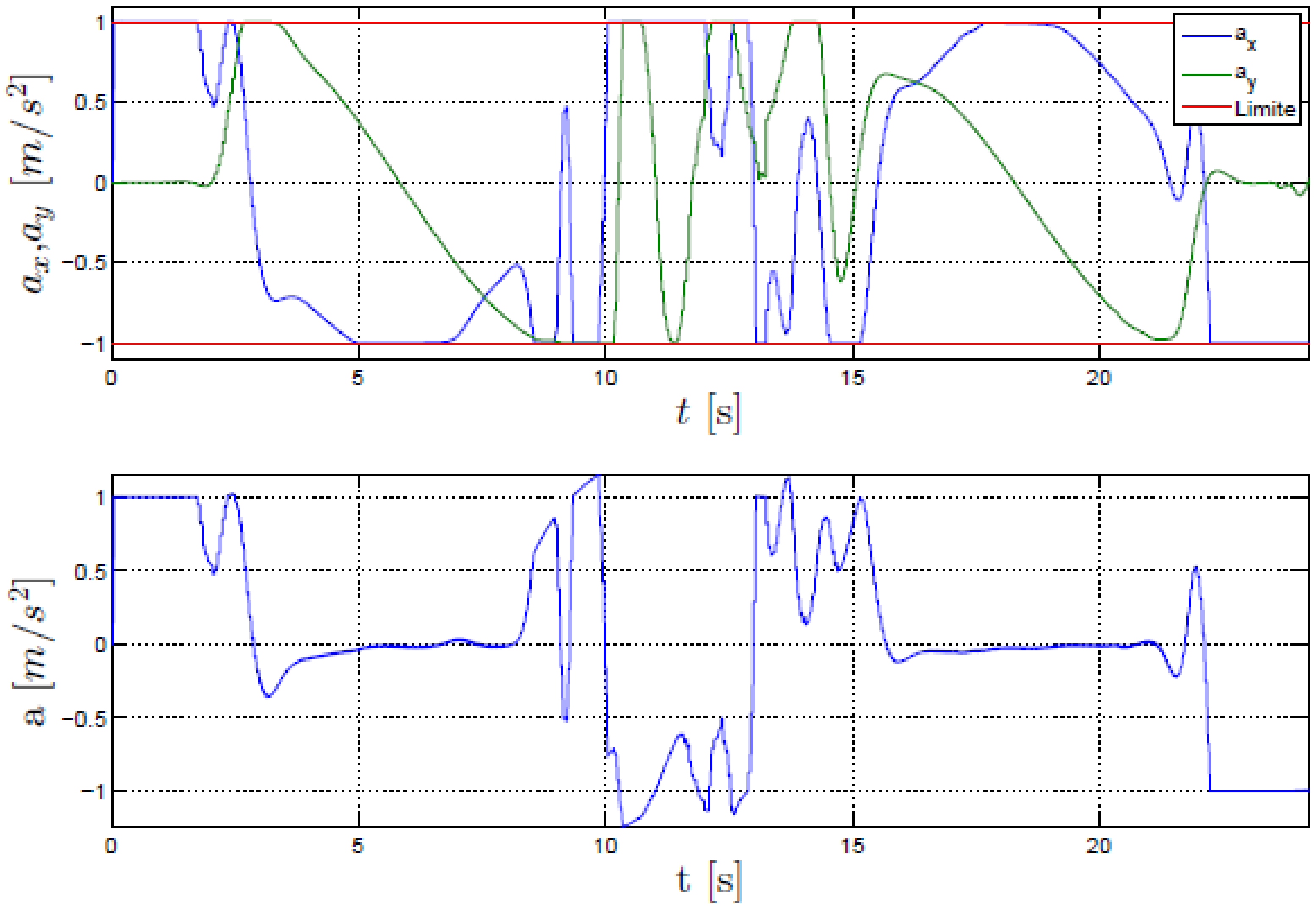

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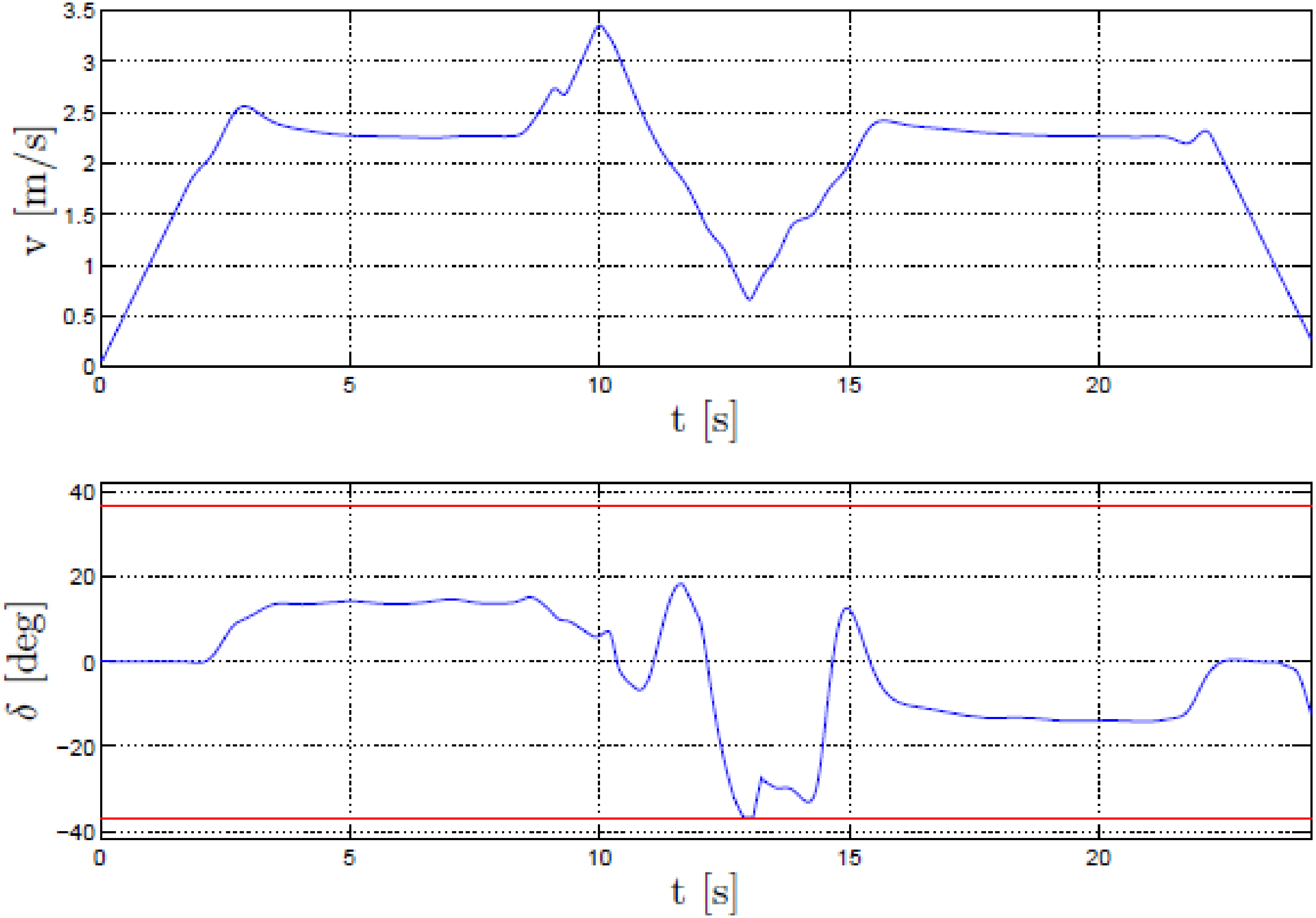
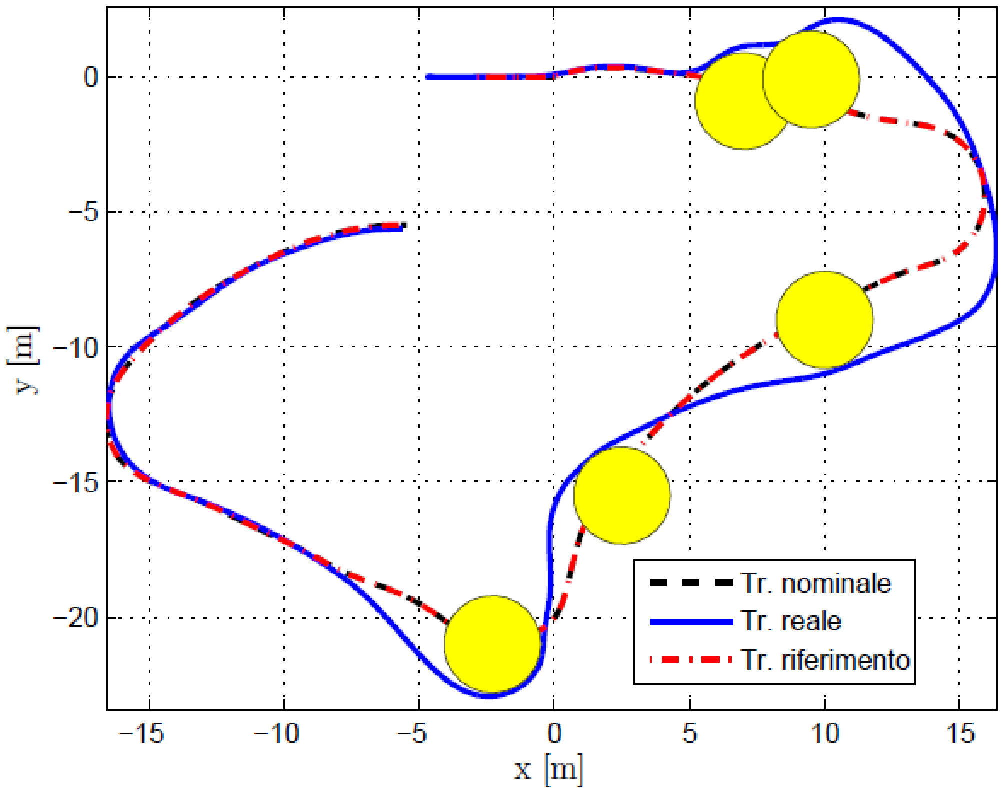


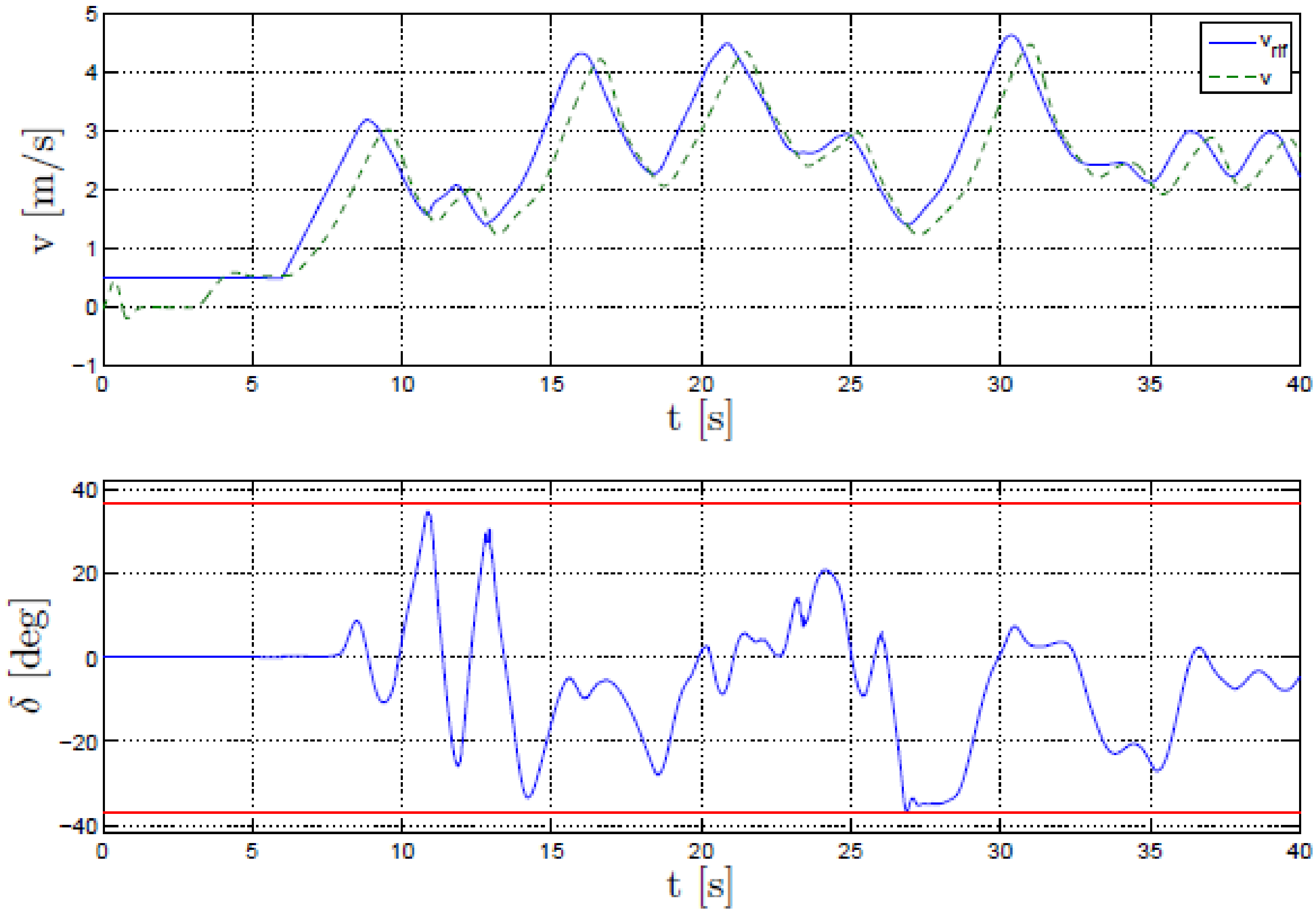
Some simulation results (Dymola vehicle dynamics simulator)

Note that:

- A simple PID-based longitudinal speed control has been implemented
- Cornering stiffness coefficients are estimated.



Some simulation results (Simulink model)



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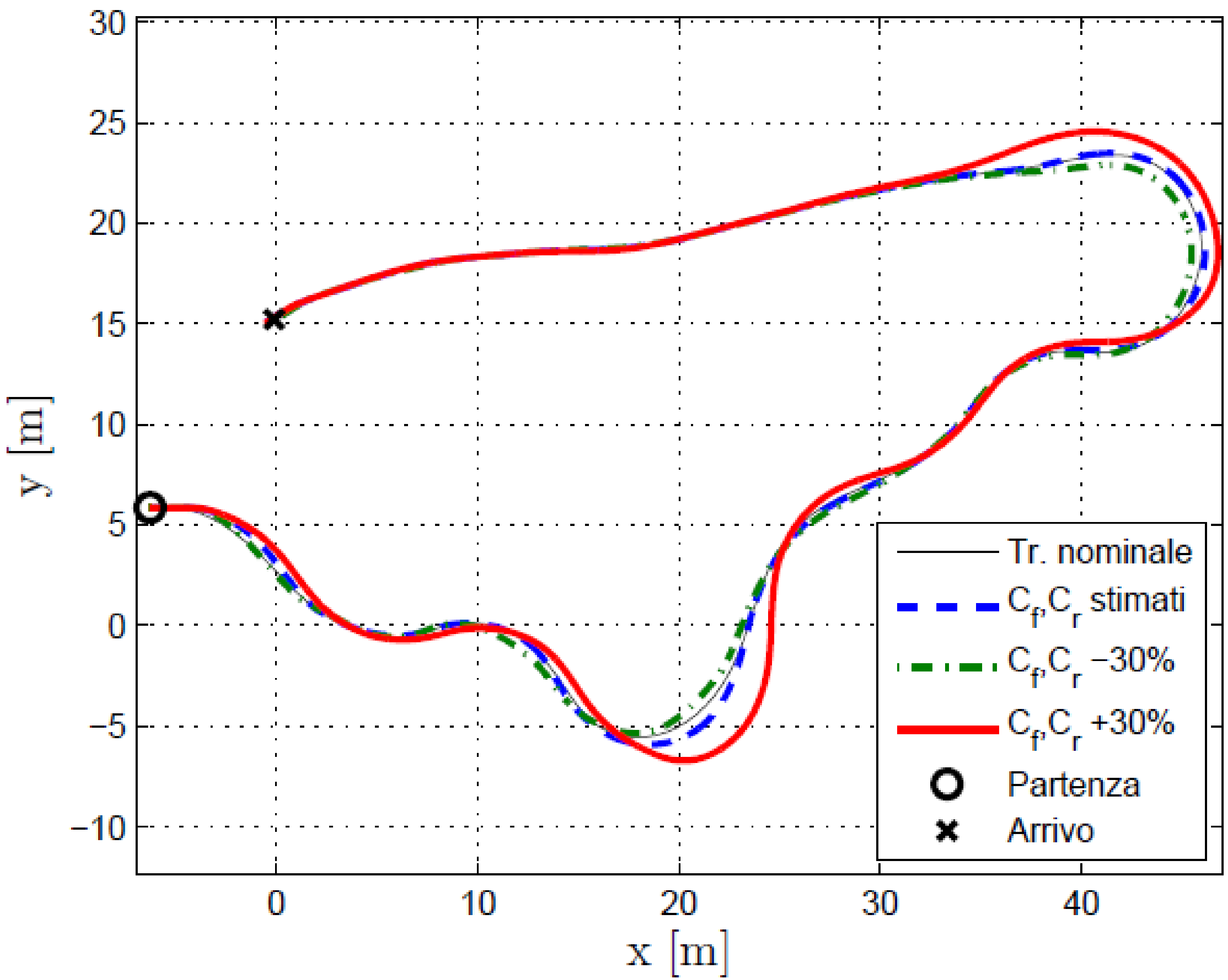
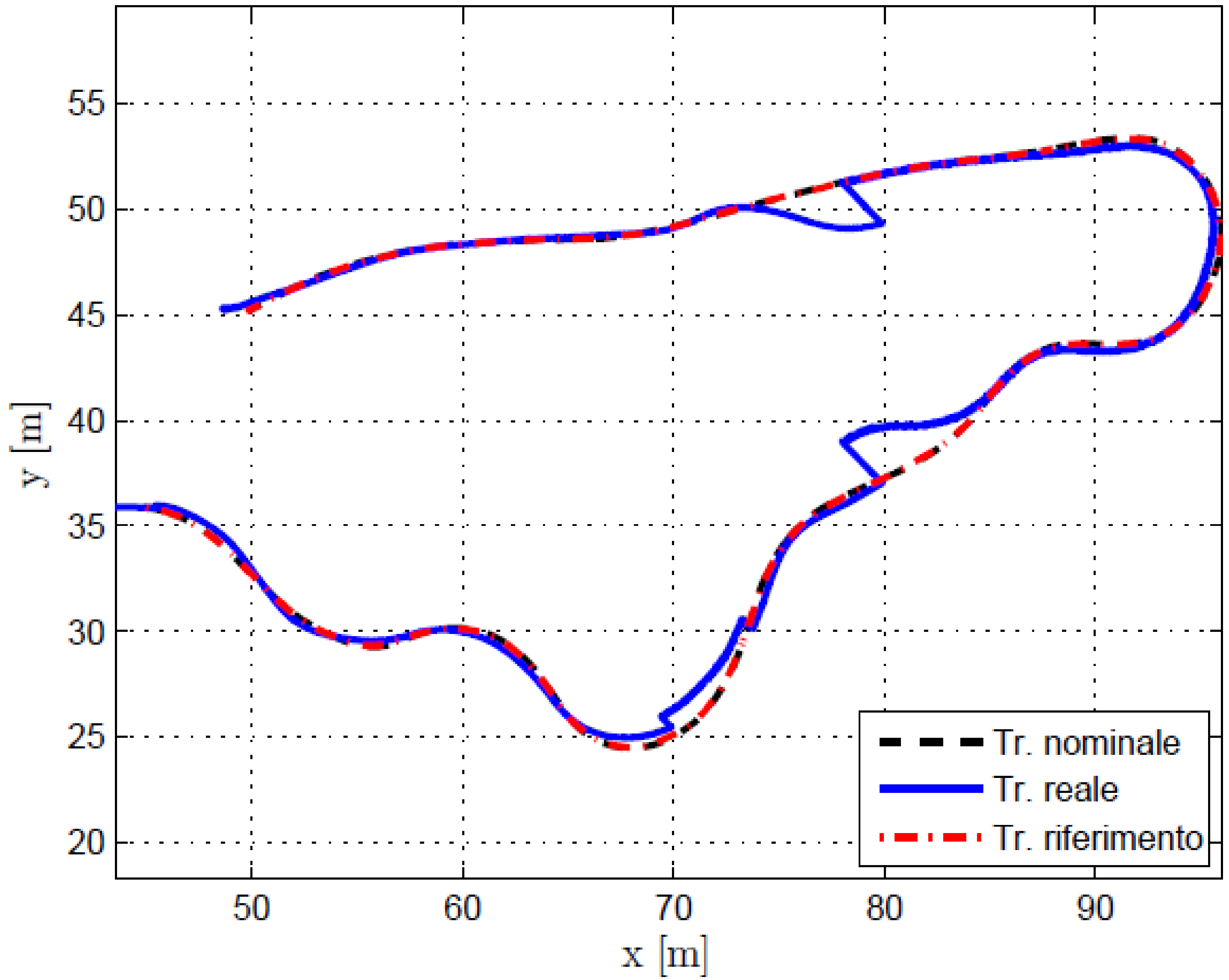
Robustness issues

Some simulations have been carried out to test the robustness of the approach (and in particular the feedback linearization, which uses precise information on the system model parameters) with respect to parameter uncertainties.

Robustness issues: Uncertainty on the cornering stifness coefficients



Robustness issues: localization errors



Robustness issues

The feedback linearization has been found to be non robust with respect to uncertainties in the position of the center of mass.

Instabilities may occur even for small uncertainties (2 cm)!!

Compensation actions are possible to make the model robust to these uncertainties

Robustness issues

In conclusion, even if a way to compensate for the lack of robustness with respect to the most critical uncertainties (position of the center of mass), further analysis and tests must be conducted to demonstrate the validity of the present approach in real cases.